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MECHANICAL ENGINEERING SCIENCE

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A Graded Course of Engineering Science

by

A. MORLEY, O.B.E., D.Sc. and E. HUGHES, Ph.D., D.Sc.

First Year

ELEMENTARY ENGINEERING SCIENCE

Second Year

MECHANICAL ENGINEERING SCIENCE

ELECTRICAL ENGINEERING SCIENCE

MECHANICAL ENGINEERING SCIENCE

A SECOND YEAR COURSE

BY
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PREFACE

The subject of this book forms part of what, in Great Britain, is called a second year part-time senior course in Mechanical or Electrical Engineering.

It is assumed that an earlier course of study has included the elements of statics and heat following some junior or preparatory course which includes some elementary Physics.

As in our earlier volume ("Elementary Engineering Science") which includes the elements of Mechanics, Heat and Electricity, emphasis is laid on energy. This not only serves as a link between mechanics and heat but also prepares the way for a simple treatment of the kinetics of gases.

The approach to the subject of heat engines has long been influenced by the former dominant position of the steam engine. A descriptive account of steam engine and boiler details has occupied a large place in the syllabuses of elementary classes and has preceded a foundation of the principles involved in the transformation into mechanical energy of the heat energy derived from combustion. Thermodynamics has been regarded as a subject of relatively advanced study, partly because it has generally involved applied mathematics beyond the range of the elementary student ; often this mathematical treatment obscures the physical reality. But this mathematical knowledge is not essential and we have endeavoured to present the elements of thermodynamics from mechanics and heat using the simple conception of the ideal gas which is adequate for an elementary study of the principles of heat engines. We believe this will give an interest to the simple gas laws and make clear what occurs in the cylinder of a heat engine, compressor or refrigerating machine.

It seems desirable that such a simple survey of fundamental principles should precede a study of details of the very numerous types of heat engines, boilers and accessories. Electrical engineering students, in courses leading to the Ordinary National Certificate, do not normally pursue the subject of heat engines beyond this stage and for them particularly a knowledge of the general principles is preferable to a study of the mechanical details relating to a small part of the field.

Fortunately technical colleges and schools are generally free to frame and revise their own syllabuses and so to advance from time to time in their methods of presentation and programmes of study. It is hoped that this book may be a contribution to such advances, involving in some cases a little recasting of syllabuses with a change in order rather than in total contents.

We wish to express our thanks to numerous friends who have made suggestions and given generous help in the preparation of this book. Particularly to Dr. Denis Taylor, who has tried and reported upon a novel presentation of the fundamental principles of heat engines and has checked numerical examples. To Principal D. N. Griffiths for help in the preparation of diagrams and in other ways, to Messrs. Babcock and Wilcox, Ltd., for a diagram (Fig. 73), and to Dr. F. T. Chapman, H.M.I., for valuable suggestions and criticism.

A. M.
E. H.

NOTE ON ABBREVIATIONS AND SYMBOLS.

These are generally in accordance with the recommendation of the British Standards Institution. (B.S. No. 560.)

THERMAL UNITS.

The British Thermal Unit (pound-degree-Fahrenheit) has been adopted as the thermal unit in general use by engineers in Great Britain and the United States. While the pound-degree-Centigrade, used by certain examining bodies has to be known by some students for examination purposes, the B.Th.U. is deliberately given preference in the steam table and elsewhere.

INTRODUCTION

As this book represents the second year's work in the subject it is desirable to state first what the student is expected to know of mechanics and heat as a result of a first year course or some other form of preparation, and to indicate what complementary knowledge he may be expected to acquire concurrently in a laboratory.

Summary of First Year Work in Mechanics and Heat.—(Quantitatively, at least, the mechanics is confined to statics.)

The equality of action and reaction as exemplified in statics.

Conservation of Energy.—No loss in transformations.

Work.—As force multiplied by distance through which it is exerted, one estimated in the direction of the other, or P . cos θ, work is associated with the transmission of mechanical energy and is a measure of the energy transmitted, and often transformed. An equivalent expression in the case of rotation about an axis is torque multiplied by angular movement, or T . θ.

The Principle of Work.—When frictional or other losses are absent, or negligible, all the energy put into a machine, measured by the work done by an effort or driving force, is transmitted and an equal amount of energy is got out of it as measured by the work done against some load or resistance. The principle may be applied to small (virtual) motions of machines, or even of structures, when such motion does not actually take place. It thus serves to give the numerical relation between the various forces exerted on a structure in equilibrium.

Moments.—For a body to be in equilibrium it is necessary that, about any axis, the total clockwise moment of forces acting upon it shall be equal to the contra-clockwise moments. This can be applied to finding an unknown force or a resultant.

Vectors.—Forces can be represented in magnitude and direction by vectors, which can be used for finding the resultant or equilibrant of several forces. If a body is in equilibrium when three forces are exerted upon it, they must be either parallel or concurrent. If concurrent, their magnitudes must be proportional to the sides of a triangle which are respectively parallel to the three forces. If several non-parallel forces maintain a body in equilibrium, lines drawn parallel and proportional to the respective forces form a closed polygon.

Forces can for convenience be regarded as the sum of two mutually perpendicular components. The sum of the vectors representing the components is equal to the vector representing the whole force.

Inclined Plane and Friction.—The force required to move a body on a smooth incline can be found by resolution of the weight into component forces or by a vector force diagram or by the principle of work. On a rough surface the reaction of the surface on any body sliding over it can be inclined to the normal at a limiting angle. The tangent of this angle is the coefficient of friction, i.e. the ratio of the frictional force along the surface to the pressure perpendicular to the surface.

Heat.—Quantity and units. Specific heat. Heat transfer in mixtures illustrated by examples; water equivalent. Calorimeters. Mechanical equivalent of heat; 778 ft.-lb. of mechanical energy converted to heat produce one British Thermal Unit, i.e. the average amount of heat required to warm 1 lb. of water 1° F. in the range from freezing point to boiling point. Latent heat, particularly of steam, sensible heat of water, boiling point and its variation with pressure and total heat of steam.

Laboratory work.—The work in this book has been deliberately limited to the science and its applications; mechanical details and descriptive matter have been purposely excluded. The illustrations are almost entirely diagrammatic and such as a student can quickly draw to make clear his own written work. In lectures it is possible to exhibit, and demonstrate the action of, mechanisms such as valves, indicators, and carburetters. And in a laboratory it is possible for the learner to examine, handle and operate experimental apparatus, models, and engine and boiler auxilliaries. These differ widely and their use is necessarily limited to the samples available. The lectures, the laboratory work, and perhaps the machine construction class, will, however, by the use of models, exhibits, and diagrams and drawings, cover those portions of a syllabus which involve detailed descriptive knowledge. But this can be confined at this stage to what is necessary for the understanding of engines, auxilliaries or apparatus, the principles of which are dealt with in the classroom or the textbook.

In addition to descriptive work, individual laboratory experiments and class demonstrations should accompany the teaching of the science. Class demonstration will often be found more economical of time and in some cases more effective in imparting knowledge, but the proportion of individual experiment and class demonstration is a matter for decision by the teacher and upon which opinions will naturally differ. The following represent the topics on which some form of laboratory work may well be done.

Acceleration of a trolley.
Acceleration of a body falling freely.
Atwood's machine.
Acceleration of a flywheel.
Centrifugal force.
Coefficient of discharge of an orifice.
Force exerted by a jet of water.
Relation of temperature and pressure of steam.
Dryness fraction of steam.
Calorific value of fuels.
Construction and use of an engine indicator.
Determination of i.h.p. and b.h.p. of an engine (class experiment).
Operation and setting of a simple slide valve.

Finally, it is believed that the use of a reliable and fairly concise text book, unencumbered with descriptive matter, may save much class time commonly spent in taking notes. If such a book is made an integral part of class instruction the student will know where to find any point he wishes to study, or to revise, and it can very conveniently replace manuscript notes. Such a method offers a chance of improving instruction in part-time classes where time is often severely restricted.

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CHAPTER I

KINEMATICS

1. Dynamics. It is assumed that the reader has some knowledge of statics, the mechanics of bodies at rest, or persisting in unaltered motion, under the influence of a balanced system of forces.

The branch of mechanics now to be studied is that concerned with the motion of bodies resulting from the exertion of forces upon them, not in general forming a balanced system but having some resultant force or torque or both.

This part of the subject is commonly called "Dynamics." But this term may well include statics, and when it is so used the portion relating to the mechanics of bodies in motion is called "Kinetics." Probably this is a better use of terms, but the reader should be aware that "Dynamics" is often used as restricted to the branch of the subject concerned with motion under the action of force (i.e. as an alternative to Kinetics).

It is convenient first to consider the subject of motion without regard to the forces causing a body to move, and this smaller branch of the subject is called "Kinematics." Since different parts of a body may have different motions, it will simplify a preliminary study of motion to consider the motion of a *point* only.

2. Displacement. When a point has motion, its displacement is its change of position. If we are concerned only with motion in a straight line, a displacement may be sufficiently specified by a distance in feet or other linear units. But more generally, when motion is not restricted to a straight line, a displacement of a point will be specified by (*a*) the distance, in a straight line, between its initial and final positions; and (*b*) the direction of that straight line, e.g. in Fig. 1, if a point initially at A moves to C, the displacement is AC. Notice that this displacement is not dependent on the time taken, nor upon the path followed between A and C. The point may have travelled from A to B and then from B to C, or by the curved path indicated in Fig. 1, but its displacement is in each case AC. Such a diagram as Fig. 1, called a space or position diagram, can be drawn full size or, like a map, to any convenient scale.

3. Vectors. A vector is a straight line having definite length and direction but not position in space. Vectors were used in an earlier course[1] in statics to represent forces, but evidently they can also represent a displacement in magnitude and direction. Thus in Fig. 2, a displacement AC from A to C can be represented by a vector *ac* drawn parallel to AC and of a length to represent AC to some definite linear scale such as 1 inch to 1 foot.

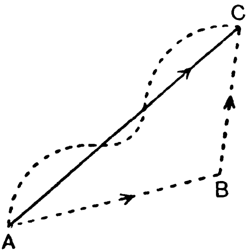


FIG. 1. Displacements.

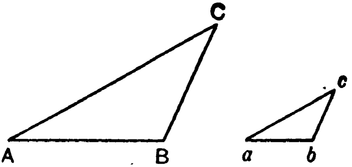


FIG. 2. Vectors.

4. Addition of Vectors. Suppose a man at A, Fig. 2, walks 4 miles due east to B and then turns into a north-east direction and walks 3 more miles from B to C. His displacement is AC; it is actually the sum of two displacements: AB due east and BC north-east, which might be called component displacements, the resultant displacement AC being the sum of the components AB and BC. And if vectors *ab* and *bc* be drawn parallel to AB and BC respectively and representing AB and BC respectively to the same linear scale, evidently *ac* represents the resultant displacement to the same scale as that for the components.

---

[1] *Elementary Engineering Science* (Morley and Hughes).

Thus vectors are added, like displacements, and we may write:

$$ab + bc = ac.$$

In order to subtract one vector from another it is only necessary to reverse its direction (i.e. to change its sign) and *add* it to the other vector. Thus:

$$ac - bc = ac + cb = ab.$$

Or if a man having walked (by any route) from A to C then walked back to B, his displacement from A would be

$$\mathrm{AC} - \mathrm{BC} = \mathrm{AC} + \mathrm{CB} = \mathrm{AB}.$$

5. Speed and Velocity. *Speed*, as the rate of movement of a body, of a point or even a wave motion, is widely known and understood. It does not depend upon the direction of the movement.

A body has constant speed if it moves over equal distances in all equal intervals of time, however short.

If a motor car travels 30 miles in 1 hour its *average* speed in that hour is 30 miles per hour or 44 feet per second or 2640 feet per minute. But unless it travels just 44 feet in every second, 4·4 ft. in every 0·1 sec., and so on, its speed is not constant but variable. The average speed of a body, in any time or over any distance, is the total distance divided by the time taken. Any units of length and of time may be employed, but the commonest are miles per hour and feet per second.

If a body, not necessarily moving at constant speed, is said to have at any instant a certain speed such as 30 miles per hour, or 44 ft. per sec., this means that, if its speed ceased to change, it would move 44 ft. in the next second, 88 ft. in the next 2 sec., and so on. Alternatively the actual speed of a body at any instant is its average speed during an indefinitely small interval of time including that instant.

Velocity is the speed of a body in a definite direction. Numerically the speed and velocity of a moving point are equal, but velocity, like displacement, is a vector quantity and not merely scalar like speed. Velocity may be called the rate of change of displacement, since displacement has direction as well as magnitude. Or it may be taken as the displacement per unit of time. Like displacement, then, it can be represented in magnitude (to scale) and in direction by a vector.

The standard symbol for velocity is v, but with a changing velocity it is usual to denote an initial velocity by u and a later velocity by v. Since speed and velocity are numerically equal, the same symbols are employed for speed as for velocity.

If a distance s ft. is covered in t sec. at a constant or at an average speed of v ft. per sec., then

$$v = \frac{s \text{ ft.}}{t \text{ sec.}} = \frac{s}{t} \text{ ft./sec.}$$

If the distance s is in a straight line we may also call v the average *velocity*.

We may also note that

$$s = v \text{ ft./sec.} \times t \text{ sec.} = vt \text{ ft.}$$

and
$$t = \frac{s \text{ ft.}}{v \text{ ft./sec.}} = \frac{s}{v} \text{ sec.}$$

Example. While a train is travelling at a constant speed of 45 miles per hour, (*a*) what is its speed in feet per second, (*b*) how far will it travel in 7 sec., and (*c*) how long will it take to cover 4 miles?

$$\begin{aligned}\text{Distance moved per hour} &= 45 \text{ miles} \times 5280 \text{ ft./mile}\\ &= 45 \times 5280 \text{ ft.}\end{aligned}$$

$$\text{Speed (in ft./sec.)} = \frac{45 \times 5280 \text{ ft.}}{60 \times 60 \text{ sec.}} = 66 \text{ ft./sec.}$$

$$\begin{aligned}\text{Distance travelled in 7 sec.} &= 66 \text{ ft./sec.} \times 7 \text{ sec.}\\ &= 462 \text{ ft.}\end{aligned}$$

Time to travel 4 miles or 4×5280 ft.

$$= \frac{21{,}120 \text{ ft.}}{66 \text{ ft./sec.}} = 320 \text{ sec. or 5 min. 20 sec.}$$

6. Acceleration. When a point moves with increasing speed it is said to be accelerated and the measure of the *rate* of increase of speed is called the acceleration. If the motion of the point is in a straight line, the change is in magnitude only and not in direction, speed being a scalar and not a vector quantity.

If a point moves with decreasing speed it is decelerated or retarded and the *rate* of decrease of speed is called the deceleration or retardation—a negative acceleration. But the *velocity* of a moving point, particle, or body, can change not only in numerical magnitude but also in direction; it is a vector quantity and not merely scalar. The rate of change of velocity is also called acceleration, but in this case it is a vector quantity having direction as well as magnitude. We shall return to this point in Art. 12.

In the case of change of speed in any given period of time, this is found by subtracting the initial speed from the final speed and the result is the gain in speed, which will be positive if there has been acceleration and negative if there has been deceleration. The average *rate* of change of speed is this gain (positive or negative) of speed divided by the time during which the change took place. For example, if a point is moving with a speed of 6 ft. per sec. and 4 sec. later it is moving in the same direction at 20 ft. per sec.,

its average acceleration = average rate of gain of speed

$$=\frac{20-6}{4}=\frac{14 \text{ ft./sec.}}{4 \text{ sec.}}$$

$$=3{\cdot}5 \text{ feet } \textit{per second per second},$$

which may be written 3·5 ft. per sec. per sec. or $3{\cdot}5\,\frac{\text{ft.}}{\text{sec.}^2}$.

This increase of speed may be at a constant rate of 3·5 ft. per sec. per sec. or the rate may be variable. If the acceleration is constant, the velocity is,

after 1 sec., $6+3{\cdot}5$ or $9{\cdot}5$ ft./sec.
after 2 sec., $9{\cdot}5+3{\cdot}5$ or $6+(3{\cdot}5\times2)=13{\cdot}0$ ft./sec.
after 3 sec., $13{\cdot}0+3{\cdot}5$ or $6+(3{\cdot}5\times3)=16{\cdot}5$ ft./sec.
after 4 sec., $16{\cdot}5+3{\cdot}5$ or $6+(3{\cdot}5\times4)=20$ ft./sec.

and so on, and after t sec. the velocity would be $6+3{\cdot}5t$ ft. per sec.

If the acceleration is constant and equal to f ft. per sec. per sec., the gain of speed in t sec. will be $t \times f$ ft. per sec., for there will be a gain of f ft. per sec. in each second, and if the initial speed is u ft. per sec. the final speed, v say, will be

$$v = u + f.t \text{ ft./sec.}$$

7. Graphs relating Distance, Time, and Speed. The relationships of these quantities to one another can be well exhibited by simple graphs.

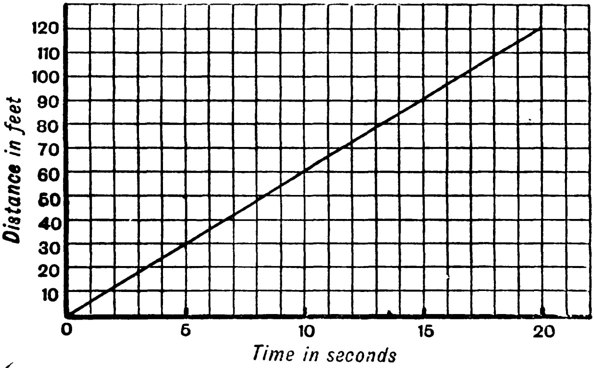


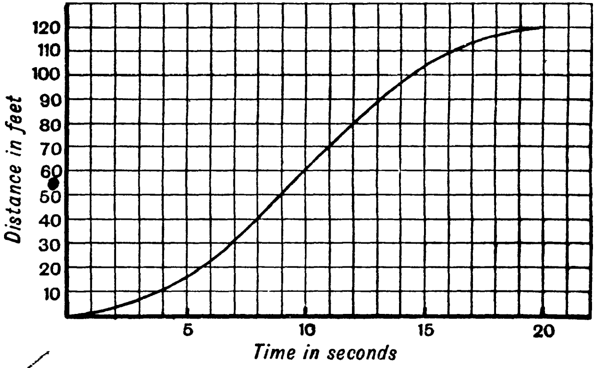
Fig. 3. Distance-time graph for speed of 6 ft. per sec.

Distance-Time Graph for Constant Speed. Fig. 3 shows the graph of distance moved at all times during a period of 20 sec. by a point moving with a constant speed of 6 ft. per sec. Wherever we measure the distance moved and the corresponding time taken we should find the number of feet moved per second to be the same, viz. 6. For example, in 15 sec. the distance traversed is 90 ft. and the average and the actual constant speed is

$$\frac{90 \text{ ft.}}{15 \text{ sec.}} = 6 \text{ ft./sec.}$$

Dividing any vertical height of the graph, representing distance, by the corresponding horizontal length, representing the time taken, gives the slope or gradient of the straight-line graph. Thus the *speed* is proportional to the *gradient* of the graph.

Distance-Time Graph for varying Speed. The same relation is true if the speed is not constant. Fig. 4 shows the graph of a point moving with varying speed. It covers 120 ft. in 20 sec. at an *average* speed of 6 ft. per sec. Also it is 6 ft. per sec. in the first 10 and in the last 10 sec. But note that the gradients, and therefore the speeds, are zero at the beginning and end of the period. It starts from rest and comes to rest at the end of 20 sec. The gradient and speed are greatest half way. Also in 2 sec. from the end of 9 sec. to the end of 11 sec. the point moves $70-50=20$ ft. Its average velocity is $\frac{20}{2}=10$ ft. per sec. which is, as nearly as it is possible to tell, its actual velocity at the end of 10 sec. It is worth noticing that in neither half of the period is the average speed (6 ft. per sec.) equal to the arithmetic mean of the greatest (10 ft. per sec.) and least (zero), namely, 5 ft. per sec. The true average is found by dividing the distance moved by the time.



✓ FIG. 4. Distance-time graph for a varying speed.

✓*Speed-Time Graph for Constant Speed.* Fig. 5 is a graph representing a constant speed of 6 ft. per sec. over a time of 20 sec. It is merely a horizontal line at a height representing 6 ft. per sec. and is only shown to introduce

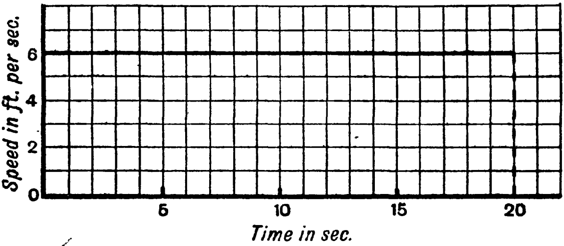


FIG. 5.Speed-time graph for constant speed.

the principle that the area enclosed under the line represents

$$20 \text{ sec.} \times 6 \text{ ft./sec.} = 120 \text{ ft.}$$

namely, the distance travelled, or in the previous general terms

$$s = t \text{ sec.} \times v \text{ ft./sec.} = vt \text{ ft.}$$

Speed-Time Graph for a varying Speed. If the speed

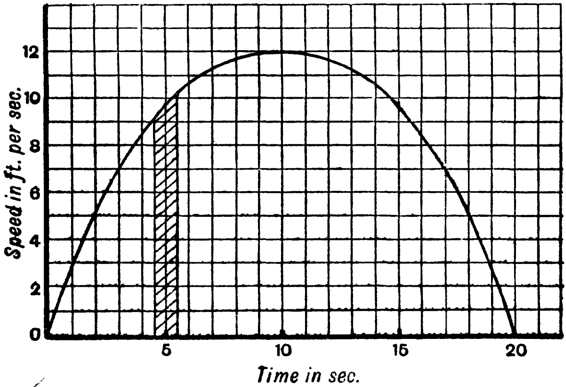


FIG. 6. Speed-time graph for a varying speed.

is not constant but varies, not in any simple way, but in a way which can be shown on a graph, the area enclosed under the graph represents the distance travelled, and calculation of this area may be the best way of determining the actual distance. For example, in Fig. 6 the speed of a moving point rises from zero initially to 12 ft. per sec. in 10 sec. and falls to zero again in another 10 sec. Consider the speed after 5 sec.; it is approximately 9·75 ft. per sec. and in the one second, from 4·5 sec. to 5·5 sec., the moving point travels 9·75 ft. which is represented by the shaded area or 9·75 ft. per sec. ×1 sec. = 9·75 ft. Similarly for all the thin strips into which we might divide the area under the graph.

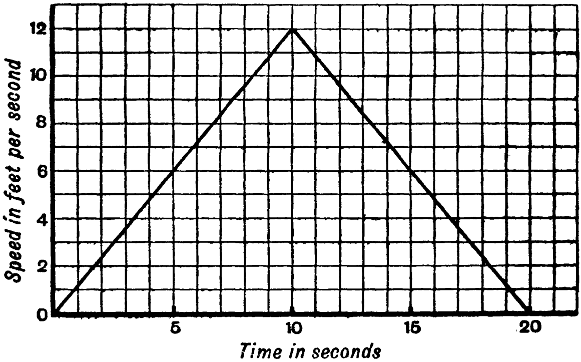


FIG. 7. Speed-time graph for speed changing at constant rate.

It is an instructive exercise to find the travel in each second and hence to plot the distance-time graph. The scale on which the area in Fig. 5 or Fig. 6 would represent the distance moved, would be 1 sq. in. to a number of feet which would be the product of the number of seconds to 1 in. on the horizontal scale, and the number of feet per second to 1 in. on the vertical scale. But generally this graphical method would resolve itself into finding the average speed from the average ordinate. This would be approximately the average of a reasonably large number of ordinates, a large number giving a more accurate result than a small number. The average speed in each half of the period in Fig. 6

is evidently considerably greater than 6 ft. per sec., which is the arithmetic mean $\left(\frac{0+12}{2}\right)$ of the greatest and least speeds, which is the true average speed for *constant* acceleration, Fig. 7, to which we now proceed.

Speed-Time Graph for Speed Changing at a Constant Rate. In Fig. 7 where the speed changes at a constant rate the calculation of the area under the curve is simple. The true average height is half the maximum or the average speed is 6 ft. per sec. and the distance travelled by the moving point is

$$6 \text{ ft./sec.} \times 20 \text{ sec.} = 120 \text{ ft.}$$

and for the period from 5 sec. to 10 sec. the average speed is

$$\frac{6+12}{2} = 9 \text{ ft./sec.,}$$

and the distance moved in this period is

$$9 \text{ ft./sec.} \times 5 \text{ sec.} = 45 \text{ ft.}$$

8. Constant Acceleration. This change of speed at a constant rate is a case of the constant acceleration referred to at the end of Art. 6. The gain of speed indicated in Fig. 7 is evidently 12 ft. per sec. in 10 sec., or 6 ft. per sec. in 5 sec., or 1·2 ft. per sec. per sec., a *constant* rate of increase or acceleration in the first 10 sec. and the rate of *decrease* or deceleration is also 1·2 ft. per sec. per sec. in the second period of 10 sec. (10 to 20). We have found the distance travelled for a selected period. Let us find it in symbols for any constant acceleration f. Let u be the speed in feet per second at

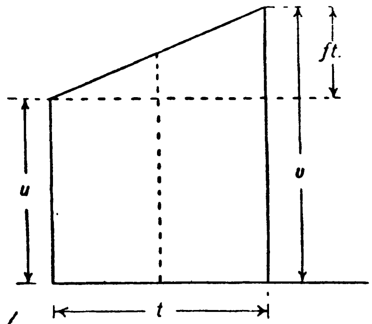


FIG. 8. Speed-time graph for constant acceleration.

any instant and let v be the speed in feet per second after t sec. Then, as in Art. 6 :

$$v=u+ft \quad . \; . \; . \; . \; . \; . \; . \quad (1)$$

The initial speed u and the final v and the gain ft are shown on the graph Fig. 8. The area under the graph represents the distance covered. The true mean height represents :

$$\frac{u+v}{2}$$

since the graph is a straight line. The distance travelled by the moving point in the time t sec. is therefore given by :

$$s=\frac{u+v}{2}\times t$$

or since $v=u+ft$, the distance is :

$$s=\tfrac{1}{2}(u+u+ft)t=ut+\tfrac{1}{2}ft^2 \quad . \; . \; . \quad (2)$$

Of these two terms, the first, ut, is the distance the point would have moved if not accelerated and $\frac{1}{2}ft^2$ is the distance it would move due to the acceleration if it had no initial speed, that is, if u were zero.

It is convenient here to note that since

$$\begin{aligned} v&=u+ft \\ v^2&=u^2+2uft+f^2t^2 \\ &=u^2+2f(ut+\tfrac{1}{2}ft^2) \end{aligned}$$

and substituting, from (2), the value s for $ut+\frac{1}{2}ft^2$,

$$v^2=u^2+2fs \quad . \; . \; . \; . \; . \; . \quad (3)$$

an expression for v^2 in terms of the initial velocity u, the acceleration f and the distance s without reference to the time t.

Starting from Rest. It is worth noticing in the very common case of a body starting from rest, that is with an initial speed $u=0$, the distance travelled by the moving point is:

$$s=\frac{v}{2}t$$

and
$$v=\frac{2s}{t}, \quad t=\frac{2s}{v}$$

where v is the maximum speed which is attained in t sec. and is also equal to ft, so that

$$s=\tfrac{1}{2}ft^2 \quad \text{and} \quad v^2=2fs \qquad . \ . \ . \ . \quad (4)$$

or
$$v=\sqrt{2fs} \qquad . \ . \ . \ . \ . \ . \quad (5)$$

which are merely particular cases of (2) and (3) above when $u=0$. They apply equally when a body comes to rest from a speed v ft. per sec. with a constant deceleration f ft. per sec. per sec.

Example 1. A train travelling at 15 miles per hour increases its speed to 45 m.p.h. in 25 sec. Find its acceleration and the distance moved by the train during the time, assuming the acceleration to be constant.

Gain of speed in 25 sec. $=45-15=30$ m.p.h.

$$\text{Acceleration } f=\frac{30 \text{ m.p.h.}}{25 \text{ sec.}}=1{\cdot}2 \text{ miles per hour per sec.}$$

which is a common way of expressing railway accelerations.

But since 60 m.p.h. $=88$ ft./sec.

$$1 \text{ m.p.h.}=\tfrac{88}{60}=1{\cdot}467 \text{ ft./sec.}$$

$$f=1{\cdot}2 \text{ m.p.h./sec.}\times\frac{1{\cdot}467 \text{ ft./sec.}}{1 \text{ m.p.h.}}$$

$$=1{\cdot}76 \text{ ft./sec./sec.}$$

The average speed is $\frac{45+15}{2}=30$ m.p.h. $=44$ ft./sec.

Distance moved in 25 sec. $=44$ ft./sec. $\times 25$ sec $=1100$ ft.

✓ **Example 2.** An observer standing opposite the front of an electric train of 8 coaches each 50 ft. long notes that 20 sec. elapses from the instant of starting till the rear end of the train passes him. Assuming the acceleration to be constant, find (1) the speed of the train when the rear end passes the observer, (2) the acceleration in miles per hour per second.

$$\text{Length of train} = 8 \times 50 \text{ ft.} = 400 \text{ ft.}$$

$$\text{Average speed} = \frac{400 \text{ ft.}}{20 \text{ sec.}} = 20 \text{ ft./sec.}$$

$$\text{Maximum speed} = 2 \times 20 = 40 \text{ ft./sec.}$$

$$\text{Acceleration} = \frac{40 \text{ ft./sec.}}{20 \text{ sec.}} = 2 \text{ ft./sec./sec.}$$

$$\text{or in m.p.h./sec. acceleration} = \frac{2}{1{\cdot}467} = 1{\cdot}36 \text{ m.p.h./sec.}$$

/Example 3. A train slows down from 60 m.p.h. to 10 m.p.h. in a distance of 680 yards. If the deceleration is constant find its amount and the time taken.

$$\text{Average speed} = \frac{60+10}{2} = 35 \text{ m.p.h.}$$

$$= 35 \times \tfrac{88}{60} = 51{\cdot}33 \text{ ft./sec.}$$

Time taken to cover 680 yards or 2040 ft.

$$= \frac{2040 \text{ ft.}}{51{\cdot}33 \text{ ft./sec.}} = 39{\cdot}74 \text{ sec.}$$

$$\text{Deceleration } f = \frac{51{\cdot}33 \text{ ft./sec.}}{39{\cdot}74 \text{ sec.}} = 1{\cdot}29 \text{ ft./sec./sec.}$$

9. Gradient of Speed-Time Graphs. In Figs. 6, 7, and 8 the excess of height of the graph at any point over that at an earlier time divided by the corresponding horizontal distance gives the average gradient and represents the average rate of gain of speed or the average acceleration in period chosen. For a constant acceleration the average and the actual are of course identical. For a varying acceleration the gradient, though varying, still represents the acceleration. The actual gradient at any point on the graph can be found from the average over a short interval on either side of the selected point. The gradient in Fig. 6 is of course zero—there is no acceleration.

In Fig. 6 the gradient and acceleration diminish from the initial time ($t=0$) to zero at the time of 10 sec. ($t=10$). The average acceleration over this 10-sec. period is 12 ft. per sec. per 10 sec. $=1{\cdot}2$ ft. per sec. per sec. At 5 sec. ($t=5$), it is evidently about $(10{\cdot}2-9{\cdot}2)$ ft. per sec. $\div 1{\cdot}0$ sec. $=1$ ft. per sec. per sec. And in the first second the increase in speed is about 3 ft. per sec. so that the acceleration at, say, time 0·5 sec. is 3 ft. per sec. per sec.

By similarly reading off the gain of speed in, say, each complete second, a graph of accelerations may be drawn and this forms an instructive exercise.

10. Acceleration of Falling Bodies. Bodies falling vertically under the gravitational pull of the earth quickly attain speeds at which the resistance of the air is considerable but by other indirect experiments it is known that with negligible resistance, falling bodies have an acceleration of about 32·2 ft. per sec. per sec. The precise amount varies somewhat in different places. Its value in London is more nearly 32·19 ft. per sec. per sec. or 981 cm. per sec. per sec.

The acceleration is generally given the symbol g in either system of units, and it is useful to remember that (5) of Art. 8 becomes

$$v=\sqrt{2gh} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

which is the speed of a body after falling h feet from rest.

Example. A pebble dropped from the top of a building reaches the ground in 3·8 sec. Estimate the height of the building taking $g=32{\cdot}2$ ft. per sec. per sec. Neglect air resistance.

$$\text{Maximum speed } v=32{\cdot}2 \text{ ft./sec./sec.} \times 3{\cdot}8 \text{ sec.}$$
$$=122{\cdot}4 \text{ ft./sec.}$$

$$\text{Average speed}=\frac{122{\cdot}4}{2}=61{\cdot}2 \text{ ft./sec.}$$

$$\text{Height of fall in } 3{\cdot}8 \text{ sec.}=61{\cdot}2 \text{ ft./sec.}\times 3{\cdot}8 \text{ sec.}=232{\cdot}6 \text{ ft.}$$

11. Angular Velocity. When a rigid body is turning about a fixed axis, as for instance, when a wheel or a crank is attached to a rotating shaft, all points in it follow paths which are circles the centres of which are in the axis. And in a minute, a second, or any other period, all points in the body make the same number of circuits about the axis. The number of revolutions per minute or per second is commonly called the speed of the shaft, and is generally specified, for example in engines, in revolutions per minute (r.p.m.).

But for some purposes it is more convenient to state the speed of spin about an axis of all these points in a body, not by the number of circles described per minute or per second, but by the angle described per unit of time. The angular movement is measured in radians and the angular velocity in radians per second. Suppose a point moves in a circle of radius 4 ft. at a constant linear speed of 12 ft. per sec. Then it will move 4 ft. along the circumference of its circular path in $\frac{4\text{ ft}}{12\text{ ft./sec.}}=\frac{1}{3}$ sec. And linear travel over an arc of 4 ft. of a circle of 4 ft. radius corresponds to an angular motion, or turning about an axis, of 1 radian. And since the point moves through 1 radian in $\frac{1}{3}$ sec. it will move through 3 radians per sec. and this is its angular velocity.

For motion about a single axis there is no distinction in meaning between speed and velocity since angular velocity here has not the vector characteristic of direction (for a single axis).

Taking the more general case in symbols, if a point in Fig. 9 moves in a circle of radius r ft. from P to P′ through an angle θ radians in t sec. at a constant linear speed v ft. per sec. it covers an arc $PP'=r\theta$ (or $\theta=PP'/r$) and its average linear speed $v=PP'/t$, or

$$\text{average angular velocity}=\frac{\text{angular movement in radians}}{\text{time in seconds}}$$

$$\omega=\frac{\theta}{t}=\frac{PP'}{rt}=\frac{vt}{rt}=\frac{v}{r}\text{ radians per sec.}$$

The angular velocity is denoted by the symbol ω (omega) and it has no linear dimensions, i.e. it has the same numerical value whether we measure the radius and the travel along the circumference in feet, inches, or other linear units. But both radius r and travel PP′ must be expressed in the *same* units. Then:

$$\omega=\frac{v}{r}\text{ radians/sec.}\quad\text{or}\quad v=\omega r\quad . \quad . \quad . \quad (1)$$

or in words

$$\omega=\frac{\text{linear speed}}{\text{radius from axis}}$$

Like linear speed, angular velocity can be constant or variable. If it is constant, then the average angular velocity over any period is the actual angular velocity. If it is varying, then the angular velocity at any instant is the average angular velocity for an indefinitely small time including that instant. If a body has an angular velocity of, say, 3 radians per sec. at any instant it would, if its angular velocity ceased to change, move through 3 radians about its axis of rotation in the next second or 0·3 radians in 0·1 sec. and so on.

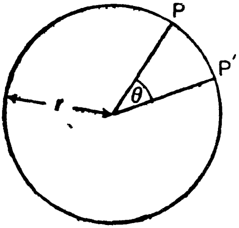


Fig. 9. Angular displacement and velocity.

Since the speed of a rotating body about an axis is sometimes expressed in radians per second and often in revolutions per minute, we should see how to convert one to the other.

In 1 revolution a body turns through 2π radians, for if it is distant r from the axis its linear travel is one circumference or $2\pi r$ and its angular travel is $2\pi r/r=2\pi$ radians. If a shaft makes N revolutions per minute, then in 1 minute the angle turned through is $2\pi N$ radians, hence the angular velocity is given by:

$$\omega=\frac{2\pi N}{60}=\frac{\pi N}{30}\text{ radians per sec.}\quad . \quad . \quad (2)$$

Angular Acceleration and Deceleration. The rate of increase of angular velocity, when positive, is called angular acceleration and when negative, angular deceleration or angular retardation. The standard symbol for this is α (alpha). Angular acceleration can be either constant or varying. If the angular velocity increases from ω_1 radians per sec. to ω_2 radians per sec. in t sec., the total increase of angular velocity is $\omega_2-\omega_1$ and if the rate of increase is constant,

$$\text{Angular acceleration, } \alpha=\frac{\omega_2-\omega_1}{t}\ \frac{\text{radians/sec.}}{\text{sec.}}$$

$$\text{or} \qquad \alpha=\frac{\omega_2-\omega_1}{t} \text{ radians/sec./sec.} \quad . \quad . \quad . \quad (3)$$

$$\text{or} \qquad \omega_2=\omega_1+\alpha t \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and if the angular displacement is θ

$$\theta=\frac{\omega_2+\omega_1}{2}\times t=\omega_1 t+\tfrac{1}{2}\alpha t^2 \quad . \quad . \quad . \quad . \quad (5)$$

$$\text{and} \qquad \omega_2{}^2=\omega_1{}^2+2\alpha\theta \quad . \quad . \quad . \quad . \quad (6)$$

formulæ exactly comparable with those in Art. 8 for constant linear acceleration.

Since $\omega=\frac{v}{r}$, if v_2 and v_1 are the final and initial linear speeds of a point moving in a circular path of radius r,

$$\alpha=\frac{\omega_2-\omega_1}{t}=\frac{v_2-v_1}{rt}=\frac{f}{r} \quad . \quad . \quad . \quad (7)$$

where f is the constant rate of increase of linear speed or linear acceleration of speed (without regard to direction).

Example. A flywheel, 5 ft. diameter, rotates at a speed of 130 r.p.m. Find its angular velocity in radians per second, the linear speed of a point on its rim, and the angular retardation, assuming it to be constant, if the wheel comes to rest in 65 sec.

$$130 \text{ r.p.m.}=130\times 2\pi \text{ radians/min.}$$

$$=\frac{130\times 2\pi}{60} \text{ radians/sec.}$$

$$=13{\cdot}6 \text{ radians/sec.}$$

Linear speed at a radius of 2·5 ft.

$$=2{\cdot}5\text{ ft.}\times 13{\cdot}6\text{ radians/sec.}$$
$$=33{\cdot}95\text{ ft./sec.}$$

$$\text{Angular retardation}=\frac{13{\cdot}6\text{ radians/sec.}}{65\text{ sec.}}$$
$$=0{\cdot}209\text{ radian/sec./sec.}$$

12. Acceleration (Vectorial). We must now return to changes of velocity as distinct from changes only in speed. In Art. 6 we considered the rate of change of speed only and deferred the rate of change of velocity which is a vector quantity. It can therefore change in direction as well as in magnitude.

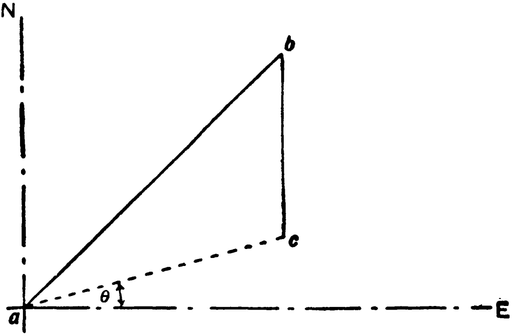


FIG. 10. Vectorial change of velocity.

If a point moving in a straight line at 15 m.p.h. changes to 20 m.p.h., its change of speed is $20-15=5$ m.p.h. Or if it changes to 12 m.p.h., its gain of speed is $12-15=-3$ m.p.h.

If a moving point changes its direction of motion it must move along a curved line, and changes in its velocity are no longer divisible into positive and negative changes; they are not merely scalar changes but vectorial changes. But it is still true that the total (vector) change in any interval of time is found by subtracting the initial velocity from the final velocity. But the subtraction is by vectors and not by arithmetic.

A vector can, by its length, represent the magnitude of a velocity (say, in feet per second) and by its direction can represent the direction of the velocity. Thus vectors can be used to find the direction and magnitude of the change in velocity of a point moving along a curve.

Suppose a body is moving due north at 4 ft. per sec. and after 3 sec. is moving north-east at 8 ft. per sec. Find (*a*) the change in velocity and (*b*) the average acceleration.

To find the change in velocity we require to subtract a velocity of 4 ft. per sec. due north from a velocity of 8 ft. per sec. north-east (i.e. 45° east of north).

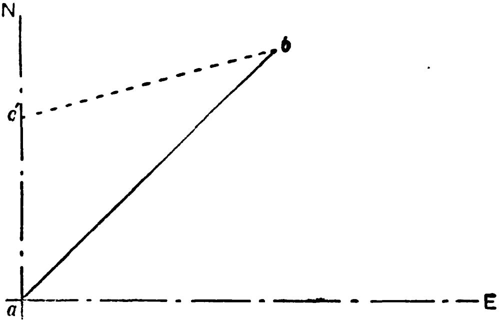


FIG. 11. Vectorial change of velocity.

From *a*, Fig. 10, set off a vector *ab* to represent 8 ft. per sec. to scale and in a direction north-east. To subtract the northward velocity, *add* a southward velocity of 4 ft. per sec. by drawing a vector *bc* in a direction south from *b*. Then

$$ac = ab + bc \quad \text{or} \quad ab - cb.$$

and this represents the change in velocity. It represents a velocity 5·9 ft. per sec. in a direction about 16° north of east.

We might look at this subtraction slightly differently by drawing the vector *ab* to represent the final velocity, Fig. 11, and the vector *ac′* to represent the initial velocity and then considering what must be added to ac' to give a result ab. Evidently it needs the vector $c'b$. And this is of course equal and parallel to ac in Fig. 10.

Vectors provide an easy method of finding velocity changes but these changes can also be calculated by the use of simple trigonometry aided by a sketch not necessarily drawn to scale. This method may be illustrated for this problem. In Fig. 12, with the same letters as in Fig. 10 but with d added, we can easily calculate the length ac.

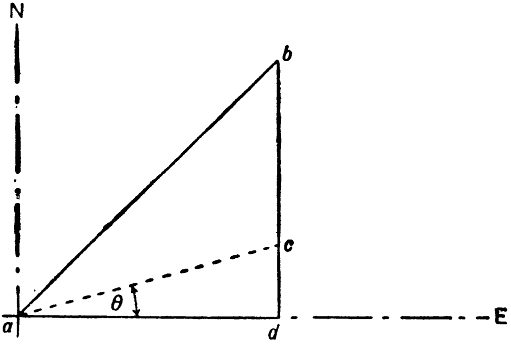


FIG. 12. Vectorial change, by trigonometry.

For

$$ad = bd = ab \cos 45° = 8 \times 0{\cdot}707 \text{ ft./sec.}$$
$$= 5{\cdot}656 \text{ ft./sec.}$$
$$cd = bd - bc = 5{\cdot}656 - 4 = 1{\cdot}656 \text{ ft./sec.}$$
$$ac^2 = ad^2 + cd^2 = (5{\cdot}656)^2 + (1{\cdot}656)^2 = 34{\cdot}74.$$
$$ac = \sqrt{34{\cdot}74} = 5{\cdot}9 \text{ ft./sec.}$$

and

$$\tan \theta = \frac{1{\cdot}656}{5{\cdot}656} = 0{\cdot}293 = \tan 16{\cdot}3°.$$

In whatever way we find it the change is the same; and since it occurs in 3 sec.,

$$\text{average acceleration} = \frac{5{\cdot}9 \text{ ft./sec.}}{3 \text{ sec.}} = 1{\cdot}97 \text{ ft./sec./sec.}$$

and is in a direction about 16° north of east.

13. Motion in a Circle. Motion of a point at uniform speed along the circumference of a circle is a good example of a change in direction. Moreover, it is of frequent occurrence and is of future use in our subject.

It will be more instructive to investigate the motion in symbols than in numbers.

Let v ft. per sec. be the constant speed with which a point moves in a circle of radius r ft. Then its angular velocity in radians per second will be

$$\omega = \frac{v}{r}.$$

Suppose we consider the changes in velocity from the time the point is at P till it is at P′, Fig. 13, having moved

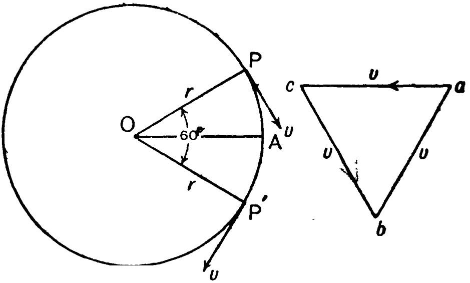


FIG. 13. Change of velocity on circular path.

through an angle of 60°. There is no change in speed, that is, in the *magnitude* of the velocity v. Instead of moving along the direction of the tangent to the circle at P it is at P′ moving along the tangent at P′. It has changed its direction by 60°. If we draw *ab* parallel to the tangent at P′ and of length to represent v to scale, then to subtract the initial velocity v we draw *bc* parallel to the tangent at P and mark off v to scale in the opposite direction to that of motion at P, i.e. from *b* to *c*, and *ac* will then give the change in velocity from P to P′. Evidently *abc* is an equilateral triangle and *ac* is equal to *ab* and *bc*, it represents a velocity v ft. per sec. in a direction AO which bisects the angle between OP and OP′. And if we divide this change of velocity

v by the time taken by the point to travel from P to P′ in seconds we can find the average acceleration during this time. The arc PP′ subtending 60° at the centre O is $\frac{1}{6}$ of the circumference $2\pi r$ ft. which the point travels at v ft. per sec.; hence the time taken to cover the arc PP′ is $\frac{1}{6}$ of $\frac{2\pi r}{v}=\frac{\pi r}{3v}$.

Hence the average acceleration $=\frac{\text{change of velocity}}{\text{time for change}}$

$$=v\div\frac{\pi r}{3v}=\frac{3\,v^2}{\pi\,r}\text{ ft./sec./sec.}$$

$$=0{\cdot}955\,\frac{v^2}{r}.$$

And if A is the circumference midway between PP′, this average acceleration between P and P′ is the approximate acceleration of the moving point in the position A. Note particularly that it is radially *inward* towards O, the centre of the circle in which the point moves.

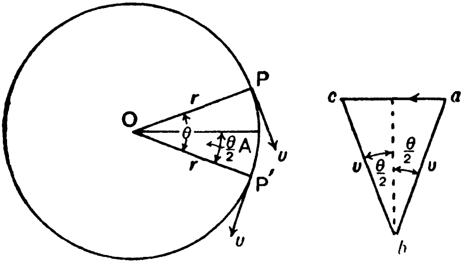


FIG. 14. Change of velocity on circular path.

If we repeat the calculation for a smaller arc of travel than PP′, but an arc of which A is still the middle point, we shall get a closer approximation to the true acceleration of the moving point when it is at A.

If we write θ instead of 60° for the angles $\widehat{\text{POP}'}$ and $\widehat{abc}$, see Fig. 14, we should see that the length of $\frac{1}{2}$ of ac is $ab\sin\frac{1}{2}\theta$ representing $v\sin\frac{1}{2}\theta$, or

$$ac=2v\sin\tfrac{1}{2}\theta\text{ ft./sec.}$$

And the time to travel through an arc PP′($r\theta$ ft.) is

$$\frac{r\theta\text{ ft.}}{v\text{ ft./sec.}}=\frac{r\theta}{v}\text{ sec.}$$

Hence,

$$\text{average acceleration}=2v\sin\tfrac{1}{2}\theta\div\frac{r\theta}{v}\text{ ft./sec./sec.}$$

$$=\frac{v^2}{r}\times\frac{2\sin\frac{1}{2}\theta}{\theta}\text{ ft./sec./sec.}$$

The fraction $\frac{2\sin\frac{1}{2}\theta}{\theta}$, which is 0·955 for $\theta=60°$, gets quickly nearer to unity as we consider smaller angles, as we can easily test by the trigonomical tables, expressing θ of course in radians (since $r\theta$ is the arc PP′).

If $\theta=30°$, the fraction is $\frac{2\sin 15°}{\pi/6}=0{\cdot}986$.

If $\theta=10°$, the fraction is $\frac{2\sin 5°}{\pi/18}=0{\cdot}9997$

and for smaller angles, the fraction approaches even more nearly to one. Thus we say that at A the acceleration is

$$\frac{v^2}{r}\text{ ft./sec./sec.}$$

in a direction from A to O.

And since $v=\omega r$, $v^2=\omega^2r^2$. So we may also write the acceleration

$$\frac{v^2}{r}=\frac{\omega^2r^2}{r}=\omega^2r\text{ ft./sec./sec.}$$

and is always from the moving point towards the centre of the circle in which it moves.

14. Resolution and Composition of Velocities and Accelerations. In an earlier course[1] we have seen that forces can be resolved into components. If a force is represented by a vector, any two vectors which have the same (vectorial) sum may represent component forces.

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[1] For example, in *Elementary Engineering Science* (Morley and Hughes), Art. 74.

But we usually choose for convenience two components in directions which are at right angles to each other. Similarly any vector quantity such as a velocity or an acceleration can be split into components. If a body is moving in a straight line in a direction, say, between north and east, it is moving north and it is moving east. If we know its velocity, i.e. its speed and direction, we can find its speed northward and its speed eastward, or in other words its component velocity north and its component velocity east.

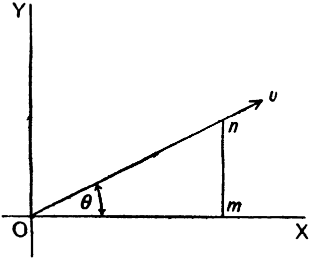

FIG. 15. Resolution of velocity.

In Fig. 15, suppose a point has a velocity represented by the vector O*n* of magnitude v and in the direction shown, inclined at an angle θ to an axis OX and $(90°-\theta)$ to OY.

Then the component velocity in the direction OX

$$om = v \cos \theta$$

and the component velocity in the direction OY

$$mn = v \cos (90-\theta) = v \sin \theta.$$

Also if we know two rectangular components of the velocity of a moving point, we can find its actual velocity by composition or combining them by the well known rule for the length of the hypotenuse of a right-angled triangle (viz. the square root of the sum of the squares of the two sides).

The purpose of dividing a velocity into two components may not at this stage be apparent, but it is useful not only as a device for making calculations but for explaining later certain principles of mechanics.

We have in effect practically used the method of resolution of velocities at the end of Art. 12. For in Fig. 12, *ad* is the eastward component of *ab* and represents 5·656 ft. per sec. (8 cos 45°) and *db* represents the northward component of *ab* and is also 5·656 ft. per sec. (or

8 sin 45°), while *cb* north and zero east are the components of *cb*. Hence subtracting the initial components from the final we have

East . . . . . . $5{\cdot}656+0=5{\cdot}656$ ft./sec.
North . . . . . . $5{\cdot}656-4=1{\cdot}656$ ft./sec.

and combining these components,

$$\text{resultant velocity}=\sqrt{5{\cdot}656^2+1{\cdot}656^2}=5{\cdot}9 \text{ ft./sec.}$$

Accelerations which are changes of velocity per second can be split into components and compounded in the same way as velocities.

15. Relative Velocity. When we speak of the velocity of a body or a point we generally mean its velocity relative to the earth, which is itself moving at high speed. That is, we think of the rate of displacement as if the earth were at rest. Similarly we sometimes speak of the velocity of one body or point relative to another which may be moving at known speed relative to the earth. At this stage we shall not need to use ideas depending on relative velocity beyond the simplest case of velocity in or parallel to a single straight line.

If a train A goes, say, east at 40 m.p.h. then relative to a second train B going east at 30 m.p.h., the first train is moving relative to the second at

$$40-30=10 \text{ m.p.h.}$$

and to an observer in the second train A would *appear* to be going 10 m.p.h. eastward, while to an observer on A, train B would appear to be going west at 10 m.p.h. But if the second train were going *west* instead of east, at 30 m.p.h. the first train A is moving east *relative* to the second train B at

$$40-(-30)=70 \text{ m.p.h.}$$

and would *appear* to be going 70 m.p.h. east to an observer in the second train B.

To put the matter briefly, the velocity of a moving point A relative to moving point B is

A's velocity minus B's velocity.

Although we shall not need to make use of it for further work at this stage, it may be stated that the foregoing statement is true also when the velocities are in any direction, but the subtraction must be made by vectors as in Art. 12, and not simply algebraically as for trains on parallel lines.

Summary of Chapter I

The displacement of a point is its change of position and has the characteristics of definite length and direction but is not dependent upon the path actually followed. A vector has the same two characteristics and can represent a displacement. Displacements can be added by vector addition.

Speed is the rate of linear movement independent of direction ; it can be either constant or varying. Velocity is speed in a particular direction and is the rate of displacement. It has the two vector characteristics and can be added, subtracted, or split into components by vector representation.

Acceleration is the rate of change of velocity and has both magnitude and direction ; it can be represented by a vector. But the term is also used for rate of change of speed. When negative, it is called deceleration or retardation. For constant acceleration,

$$v=u+ft.$$

Varying speeds and accelerations are conveniently shown on graphs. The gradient of a distance-time graph represents the speed and the gradient of a speed-time graph represents the acceleration. The area under a speed-time graph represents the distance covered, upon the scale easily found from the distance represented by one square inch of area.

Angular velocity is the rate of angular movement about an axis and is expressed in radians per second. Angular acceleration is the rate of change of angular velocity and calculations relating to it are similar to those relating to linear acceleration.

When a body moves at constant speed in the circum-

ference of a circle, its acceleration is $\omega^2 r$ towards the centre of the circle.

Vectors representing velocities and accelerations can be resolved into components representing component velocities and component accelerations ; the process is similar to that of the resolution of forces into components. Resolution into two components mutually at right angles allows arithmetic methods to be employed for adding or subtracting velocities or accelerations, as an alternative to vector representation.

EXAMPLES I

1. An aeroplane occupies 55 min. on a journey of 195 miles. What is its average speed in miles per hour and in feet per second ? How long will it take to cover 1 mile at the average speed ?

2. A pulley 2 ft. 6 in. diameter makes 240 r.p.m. Find in feet per second the linear speed of a point on the rim of the pulley.

3. If a train makes a journey of 104 miles in 2 hours, find its average speed in metres per second.

4. How long will it take a car travelling at 30 m.p.h. to come to rest with a deceleration of 8 ft. per sec. per sec. ? And how far will it travel in that time ?

5. If the speed of a body is increased from 15 ft. per sec. to 60 ft. per sec. in ½ min., find its average acceleration in feet per second per second.

6. An engine runs at 225 r.p.m. What is the angular velocity of its crankshaft in radians per second ?

7. A locomotive is travelling at 50 m.p.h. What is the angular velocity of its driving wheels if they are 4 ft. 6 in. diameter, assuming that no slipping takes place ?

8. A train starting from rest attains a speed of 60 m.p.h. in 1·5 min. Find its average acceleration in miles per hour per second and in feet per second per second. How long will it take to attain a speed of 40 m.p.h. from rest ?

9. A train increases its speed from 15 m.p.h. to 45 m.p.h. in passing over a distance of 400 ft. Assuming the acceleration to be constant, find the time taken and the acceleration in feet per second per second.

10. A wheel making 100 r.p.m. has its speed increased to 180 r.p.m. in 10 sec. Find its average angular acceleration in revolutions per minute per second and in radians per second per second.

11. A flywheel makes 80 revolutions in slowing down from 120 r.p.m. to 100 r.p.m. Assuming the angular deceleration to be constant, find the time taken and the deceleration in radians per second per second.

12. A body moves due east at 8 ft. per sec. and after 0·6 sec. is moving north-west at 16 ft. per sec. Find the average acceleration in magnitude and direction during this time.

EXAMPLES FOR GRAPHICAL SOLUTION

13. A train starting from rest, covers the following distances x ft. in the times t sec.:

t sec. . .	0	5	11	18	22	27	31	38	46	50
x ft. . .	0	10	54	170	260	390	450	504	550	570

Find the average speed during the first 10 sec., during the first 30 sec., and during the first 50 sec. Also find approximately the actual speeds after 5, 15, 25, 35, and 45 sec. from the start and plot a speed-time graph for the whole time.

14. From the results of No. 13 draw an acceleration-time graph by finding the average accelerations at intervals of 5 sec. each.

15. A train travelling at 30 m.p.h. is retarded by brakes and its speed v in m.p.h. after t sec. is given as follows:

t sec. . .	0	4	12	20	26	35	42	50
v m.p.h. .	30·0	26·0	21·5	16·7	14·0	10·4	7·7	4·8

Find the average retardations in feet per second per second at 5-sec. intervals and draw a retardation-time graph. Find the distance covered in the first 30 sec.

CHAPTER II

LAWS OF MOTION

16. Force and Motion. Having considered the displacement, velocity, change of velocity, and acceleration of a point which may be in a moving body, we approach the very important part of mechanics relating to the movement of bodies consequent upon forces exerted upon them. The three laws of motion were first put in form by Sir Isaac Newton (a great English scientist, 1642–1727), and are known as Newton's Laws of Motion.

It is a matter of common experience that when force is exerted upon a body of appreciable size the effect may depend considerably upon where the force is applied. For example, if we toss in the air a body of elongated shape and hit it with a stick it will be driven forward, but, unless it is hit centrally, it may also spin as it flies. In this chapter we want to avoid such complications and to consider bodies, every point of which has the same motion, which can be considered like that of a moving point. A sphere of very small size in comparison with any linear dimensions involved in its motion would approximate to this condition and sometimes such a body is called a material particle. With this explanation we shall in this chapter refer to the motion of a " body " on the understanding that the conditions are such that its motion is one of flight like the motion of a point and that rotation or spin about an axis can be ignored. Actually the body need not be small.

17. First Law of Motion. *A body continues in its state of rest or motion in a straight line with constant velocity except in so far as it is compelled by external force to change that state.*

This may be called the " law of inertia." A body will not change its state of motion or rest unless compelled to do so: it resists change of velocity in magnitude or direction, that is it resists acceleration.

From this law we may define force as that which when exerted upon a body tends to change its motion.

Though we know of no case of a body on which no force is exerted, it is not difficult to arrange that a body shall have exerted upon it only forces which are very nearly balanced, e.g. a steel ball rolling on a horizontal steel plate or one so slightly inclined as just to balance any frictional resistances to rolling. And in an experimental lifting machine it is possible so to adjust the effort and load as to produce an approximate balance permitting motion of all parts to continue at a constant speed, and the load to move in a straight line at constant velocity over a limited range.

18. Inertia. The property of matter, indicated in the *First Law of Motion*, of reluctance to change of motion is a matter of common experience. A heavier body requires more effort to make it change from one speed to another than a lighter body irrespective of their sizes. It exhibits more reluctance to change of speed or direction. It is more inert or it posseses more *inertia* which is the name given to the property of resisting change of speed or direction of motion.

19. Mass and Weight. While the reluctance of a body to change its motion is called inertia, when this is a measurable quantity it is called *mass*. The standard of mass is a piece of metal kept in the custody of the Board of Trade in London and is called the standard pound. Masses are therefore measured in pounds. In the centimetre-gramme-second system the unit mass is the gram which is approximately the mass of 1 cubic centimetre of water at its maximum density, namely, at 4° C. It is about $\frac{1}{453 \cdot 6}$ of 1 lb.

The usual unit of force for engineering purposes is also called the pound force. It is the gravitational pull of the earth upon one pound mass at a standard place—

London. This is often called the weight of one pound mass. But the word "weight" is used in several senses, e.g. (1) the ratio of the mass of a body to that of the unit mass, (2) the force exerted by a body on its supports when it is at rest, (3) the gravitational pull of the earth *on* the body, which is equal to (2).

In mechanics we must distinguish between mass and weight and having explained what is meant by mass, we can regard weight in the third of the foregoing senses, that is, briefly as the gravitational force on one pound mass. The usual and standard abbreviation for the word pound is lb. Various attempts have been made to allocate some different abbreviation to distinguish the pound force from the pound mass but no such plan has secured general adoption and we shall use lb. for the word pound whether it refers to mass or to force.

Although the mass and the weight of a body have different meanings, the weights of different bodies are exactly proportional to their masses. For the force of attraction of the earth on all bodies is proportional to their masses. Consequently the masses of different bodies are generally determined by weighing them in a balance: that is by comparing the forces exerted by the earth upon them with that exerted upon the standard mass. In this, of course, we use copies of the actual standard pound mass and multiples and subdivisions in the form of metal masses usually called "weights."

The density of a body ρ (rho) is its mass per unit of volume or

$$\rho=\frac{m}{V}.$$

Where m is the mass and V the volume of the body. Thus density may be stated in pounds per cubic foot.

The weight of a body, that is the force exerted by the earth upon it, differs slightly at different parts of the earth's surface. The difference is very small but we shall return to this point (Art. 23).

If we had a very sensitive and accurate spring balance, we could measure the differences in the weight of a body at different places. But if we weighed a body by balancing it against standard masses (usually called "weights"), we should detect no such differences.

The mass of a body is sometimes said to be the quantity of matter in it, being the product of the two factors volume V and density ρ. This may be helpful but the nature of mass cannot be fully appreciated without reference to its quality of inertia or reluctance to change of motion which is further dealt with in succeeding articles.

20. Relation of Force, Mass, and Acceleration. We now come to the very important section of mechanics concerning bodies which do change their state of rest or motion; that is, bodies which change their velocity, in magnitude or direction or both, or in other words, which have an acceleration. We shall approach this matter in two ways, both of which are desirable for a proper understanding of the mechanics of motion.

Firstly, it is a fact of direct and indirect experiment that the exertion of different forces on any particular body produce in it accelerations which are proportional to those forces. This can be demonstrated by a laboratory experiment on a trolley running on a track just sufficiently inclined to overcome frictional resistances. With no force other than that of gravitation on the slight incline, the trolley will run at constant speed when started. For different pulling forces, which can be arranged without change in the total mass moved, it is found that the acceleration is proportional to the force applied to the trolley.

Let W lb. be the weight of the body and g ft. per sec. per sec. be the acceleration with which it would fall freely towards the earth if dropped. That is, let g ft. per sec. per sec. be the acceleration of the body when acted upon by a force W lb.

Then if P lb. be any other force which produces an acceleration f ft. per sec. per sec., the proportionality shown by the experiment is

$$\frac{f}{g}=\frac{P}{W} \quad . \ . \ . \ . \ . \ . \ . \ . \quad (1)$$

or

$$P=\frac{W}{g}.f \quad \text{or} \quad W\frac{f}{g} \text{ lb.} \quad . \ . \quad (2)$$

This gives by a simple and direct method the force necessary to produce an acceleration f in a body of weight W whatever units are employed, provided the meanings of the symbols are correctly observed. The pound force is that with which the earth attracts in London the piece of metal which represents the standard pound mass. Statement (2) is true for any place on the earth provided P and W are in pounds as just defined. In this case, W in pounds will differ (slightly) in different places. But as g will differ in exactly the same way from place to place the value of W/g will remain constant and the force P will remain the same everywhere for any acceleration f.

We may take W to be the weight of the body in pounds in London; then g must be the acceleration or value of g in London which is about 32·2 ft. per sec. per sec.

Since $P=f\times W/g$, the factor W/g is a measure of the inertia of the body or of its reluctance to accelerate. With a suitable choice of units it represents its mass, but we shall return to this later when we deal with units in Art. 23.

21. Momentum. This is the name given to the product of the mass of a body and its velocity, or $m\times v$. The unit of momentum is that of a body of unit mass moving with unit velocity, e.g. it may be 1 lb. ×1 ft. per sec. or 1 lb.-ft. per sec. or we may adopt a unit of mass other than the pound. The momentum of a body is sometimes called the quantity of motion in it.

Momentum is evidently a vector quantity having magnitude and direction, for mass has no direction and is simply scalar, so that a vector which represents velocity in magnitude and direction can also represent, to a different scale, momentum which is the velocity multiplied by a constant mass. Hence changes in momentum can be found by vectors just in the same way as changes in velocity. Also momentum can be resolved and compounded in the same way as velocity.

22. Second Law of Motion. *The rate of change of momentum is proportional to the force exerted and takes* *place in the direction of the straight line in which the force acts.*

This is in effect a definition of force in respect of its effect on a body which it can move. It follows that a *constant* force will in any given time, say, 1 sec., generate a given amount of momentum in its own direction. If the mass m of the body upon which the force is exerted is large, the velocity gained in the given time may be small; it will be inversely proportional to the mass. That this is true can be demonstrated by the experimental trolley carrying varying masses on which a constant force is exerted in turn. The momentum gained per second will be a measure of the force exerted upon the body.

Put in symbols for a force P exerted upon a mass m moving with a variable velocity v,

$$\text{P} \propto \text{rate of change (increase) of } (m \times v). \quad (1)$$

and provided m remains constant,

$$\text{P} \propto m \times \text{rate of change of } v \ldots\ldots \quad (2)$$

or

$$\text{P} \propto m.f \ldots\ldots \quad (3)$$

where f is the acceleration of the mass m.

This proportionality can be written

$$\text{P} = m \times f \times \text{constant}$$

the value of the constant depending upon the units adopted. With a suitable choice of units we can make the constant unity or

$$\text{P} = m \times f \ldots\ldots \quad (4)$$

This will be true if we take the unit force as that which will give unit acceleration to a unit mass. This will be true of any system of units. Putting this in words,

$$\text{Force} = \text{mass} \times \text{acceleration} \ldots \quad (5)$$

or

$$\text{acceleration} = \frac{\text{force}}{\text{mass}} \ldots\ldots \quad (6)$$

23. Units of Force. If we keep to 1 lb. as the unit of mass and 1 ft. per sec. per sec. as the unit of acceleration, then in order that unit force shall be that which gives unit mass unit acceleration, we must adopt as the unit of force one which will give a mass of 1 lb. an acceleration of 1 ft. per sec. per sec. This force is called the *poundal*. Its definition needs no reference to any fixed place on the earth's surface and it is sometimes called an absolute unit of force. But it is not used in physics nor is it in general use by engineers who measure force in pounds. It is equal to about $\frac{1}{32\cdot2}$ of a pound force or roughly equal to the gravitational force on ½ oz. mass in London, for if in equation (4) of Art. 22 we write $f=32\cdot2$ ft. per sec. per sec. for a pound mass falling under the pull exerted on it by the earth in London,

$$P=1 \text{ lb. force}=1 \text{ lb. mass}\times32\cdot2 \text{ ft./sec./sec.}$$

$$=32\cdot2\,\frac{\text{lb. mass}\times\text{ft.}}{(\text{sec.})^2} \quad\text{or}\quad 32\cdot2 \text{ poundals} \quad . \quad . \quad (1)$$

or 1 lb. force is 32·2 poundals of force.

For engineering purposes it would be possible to make calculations in poundals as the unit of force; and for practical use, to convert any force in poundals to pounds of force by dividing it by 32·2. A few people do this. But it is not the only plan, for we can use the pound force as the unit of force and 1 ft. per sec. per sec. as the unit of acceleration if we use a unit of mass other than 1 lb.

Engineers' Units. 1 lb. force gives an acceleration of about 32·2 ft. per sec. per sec. in London to a mass of 1 lb. And it will give an acceleration of 1 ft. per sec. per sec. to a mass of 32·2 lb. Then the relation given in equation (4) of Art. 22, namely,

$$P=m\times f$$

still holds, and if m is the mass in pounds

$$P \text{ in poundals}=m \text{ lb.}\times f \text{ ft./sec./sec.} \quad . \quad . \quad . \quad (2)$$

$$P \text{ in pounds}=\frac{m}{32\cdot2} \text{ lb.}\times f \text{ ft./sec./sec.}$$

$$\text{or}\quad P \text{ in pounds}=\frac{W \text{ lb. force}}{g \text{ ft./sec./sec.}}\times f \text{ ft./sec./sec.} \quad . \quad (3)$$

$$P=\frac{W}{g}\times f \;\text{ lb. force} \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

Here W is the number of pounds of force which the earth exerts on the body in London and g is the rate of increase of velocity of fall in feet per second per second in London.

It has been pointed out in Art. 20 that the general relation

$$P=\frac{W}{g}.f$$

is true if W is the weight of the body in *any* place and g is the acceleration due to gravitation *at that place*, for W and g vary from place to place in exactly the same proportion as one another, but it must be remembered that W here is the number of pounds of force with which a body is attracted at the place considered and 1 lb. of force is measured by the attraction of the earth for 1 lb. mass in London.

It may be noted that the extreme variation of the acceleration of gravity and of the weight of a body as above defined is less than half of 1 per cent. at different places on the earth's surface.

If the physical dimensions of all the quantities are included in a calculation, it is probably best to remember that

$$\frac{\text{pounds force}}{32{\cdot}2\text{ ft./sec./sec.}}=\frac{\text{pounds mass}}{32{\cdot}2} \quad . \quad . \quad (4)$$

and in symbols

$$P=\frac{W}{g}.f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

may be written

$$P=mf \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

by substituting m for W/g where m stands for the mass in engineers' units one of which is equal to 32·2 lb. In engineers' units, the momentum of a body weighing W lb. and moving with a velocity v ft. per sec. is $\frac{W}{g}v$ units.

C.G.S. or Centimetre-Gramme-Second Units. In this system the unit of mass is the gramme, and the unit of force is the dyne, which is the force necessary to give a mass of 1 gramme an acceleration of 1 cm. per sec. per sec.

The weight of 1 gm. in London is about 981 dynes or $g=981$ cm./sec./sec. $\left(=\frac{981}{2\cdot54\times12}\text{ or }32\cdot2\text{ ft./sec./sec.}\right)$.

Engineers use the weight of 1 kilogram (1000 gm.) as a unit of force.

Example 1. A loaded motor car weighs 1800 lb. and its speed on a level road is raised from 20 m.p.h. to 50 m.p.h. in 10 sec. Find the constant force necessary for this apart from frictional or other resistance. Find also the horse-power at the beginning and end of the period in addition to that necessary to overcome frictional resistances.

The forward force on the car is exerted by the road on the driving wheels, which thrust backward on the road; we are only concerned with the horizontal component of the force exerted by the road on the car.

$$\text{Gain of speed in 10 sec.}=50-20=30\text{ m.p.h.}$$
$$=44\text{ ft./sec.}$$

$$\text{Acceleration}=\frac{44\text{ ft./sec.}}{10\text{ sec.}}=4\cdot4\text{ ft./sec./sec.}$$

Force to produce this acceleration

$$=\frac{1800\text{ lb. force}}{32\cdot2\text{ ft./sec./sec.}}\times4\cdot4\text{ ft./sec./sec.}$$
$$=\frac{1800\times4\cdot4}{32\cdot2}\text{ lb.}$$
$$=246\text{ lb.}$$

Speed in feet per second at 20 m.p.h.

$$=\tfrac{88}{1}\times\tfrac{20}{60}=29\cdot33\text{ ft./sec.}$$

Work done per second

$$=246\times29\cdot33=7214\text{ ft.-lb.}$$

$$\text{Horse-power}=\frac{7214\text{ ft.-lb./sec.}}{550\text{ ft.-lb./sec.}}=13\cdot1\text{ h.p.}$$

With a constant accelerating force, the horse-power will be proportional to speed, hence at 50 m.p.h.

$$\text{Horse-power}=13\cdot1\times\tfrac{50}{20}=32\cdot8\text{ h.p.}$$

Example 2. The piston and other reciprocating parts of a petrol engine, running at 3000 r.p.m., have at the beginning of the stroke an acceleration of 1640 ft. per sec. per sec. Find the force, per pound of these parts, which must be applied to them to give this acceleration.

(1) Since the forces are proportional to the acceleration which they will produce and 1 lb. force on a mass of 1 lb. gives an acceleration of 32·2 ft. per sec. per sec.,

$$\text{Force to give 1640 ft./sec./sec.} = 1 \text{ lb.} \times \frac{1640}{32{\cdot}2} = 51 \text{ lb.}$$

(2) Alternatively from (3), Art. 23,

$$\begin{aligned}\text{Force} &= \text{mass} \times \text{acceleration}\\ &= \frac{1 \text{ lb. force}}{32{\cdot}2 \text{ ft./sec.}^2} \times \frac{1640 \text{ ft./sec.}^2}{1}\\ &= 51 \text{ lb. force.}\end{aligned}$$

(3) Or from (2), Art. 23,

$$\begin{aligned}\text{Force in poundals} &= 1 \text{ lb.} \times 1640 \text{ ft./sec.}^2\\ &= 1640 \text{ lb.-ft./sec.}^2 \text{ or poundals.}\end{aligned}$$

$$\text{Force in pounds} = \frac{1640}{32{\cdot}2} = 51 \text{ lb.}$$

This exercise shows three ways of working a fundamentally important problem.

It also exemplifies the very large forces which occur in high speed machines. This force of 51 lb. for each pound of piston is exerted on the piston by the cylinder gases or the crankshaft or by the two jointly.

Example 3. If the maximum sliding resistance between the tyres of a motor car and the road is 0·8 lb. per lb. of load on the wheels, find the shortest time and distance in which a car could have its speed reduced from 50 m.p.h. to 20 m.p.h.

(It will be noted that the resistance is stated as 0·8 lb. per lb. rather than as resulting from a coefficient of friction of 0·8 between the tyres and the road. This is to allow of the possibility of a grip which is not entirely of a strictly frictional kind.)

Braking forces on brake drums cannot be effective to retard the car at a greater rate than that at which the wheels are locked and the tyres are sliding along the road surface. Hence this condition puts a limit on the rate of deceleration.

Since the limiting decelerating force is 0·8 of the gravitational force, the limiting deceleration (when the full decelerating force is operating and therefore is constant) is

$$0{\cdot}8 \text{ of } 32{\cdot}2 \text{ ft./sec./sec.} = 25{\cdot}8 \text{ ft./sec./sec.}$$

Speed reduction is (50 – 20) m.p.h.

$$= 30 \text{ m.p.h.} = 44 \text{ ft./sec.}$$

$$\text{Time required} = \frac{44 \text{ ft./sec.}}{25{\cdot}8 \text{ ft./sec./sec.}} = 1{\cdot}706 \text{ sec.}$$

$$\text{Average speed in this period is } \frac{50+20}{2} = 35 \text{ m.p.h.}$$

$$= 35 \times \tfrac{88}{60} = 51{\cdot}3 \text{ ft./sec.}$$

Distance travelled during deceleration

$$= 51{\cdot}3 \text{ ft./sec.} \times 1{\cdot}706 \text{ sec.} = 90{\cdot}7 \text{ ft.}$$

Example 4. If a train weighs 200 tons and there is a track resistance of 10 lb. per ton, what pull will a locomotive have to exert upon it if the train is accelerated at 0·5 mile per hour per sec. up an incline of 1 in 120 ?

The force which the locomotive must exert on the train to propel it may here be divided into three parts: (*a*) Force required to overcome track or frictional resistances. (*b*) Force to lift the train up an inclined plane independent of friction. (*c*) Force to accelerate a mass of 200 tons.

(*a*) Force required for track resistance

$$= 200 \text{ tons} \times 10 \text{ lb./ton} = 2000 \text{ lb. force.}$$

(*b*) By the principle of work, it follows that in lifting 200 tons through a vertical height of 1 ft. an effort exerted through 120 ft. does work equal to 200 × 2240 lb. force × 1 ft. Hence the effort required is

$$\frac{200 \times 2240 \text{ ft.-lb.}}{120 \text{ ft.}} = 3733 \text{ lb.}$$

(*c*) Acceleration of 0·5 m.p.h. per sec.

$$=0{\cdot}5\times\tfrac{88}{60}\text{ ft./sec./sec.}$$
$$=0{\cdot}73\text{ ft./sec./sec.}$$

Force to give 200 tons this acceleration is

$$\frac{(200\times2240)\text{ lb.}\times0{\cdot}73\text{ ft./sec./sec.}}{32{\cdot}2\text{ ft./sec./sec.}}=10{,}150\text{ lb.}$$

Total pull required $=2000+3733+10{,}150=15{,}883$ lb.

Example 5. If the train in Example 4 is travelling at 30 m.p.h. up the incline and steam pressure or other driving force is cut off, how long will it take the train to come to rest without the use of brakes?

The resistances causing deceleration are the track resistance and gravity. Both are proportional to the weight of the train, which can be considered with or without the locomotive.

If W lb. be the weight of the train and P lb. be the decelerating force,

$$P=\frac{10}{2240}W+\frac{1}{120}W=0{\cdot}004464W+0{\cdot}008333W=0{\cdot}01280W$$

$$\text{Deceleration}=\frac{P}{W}\times32{\cdot}2\text{ ft./sec./sec.}$$
$$=0{\cdot}01280\times32{\cdot}2\text{ ft./sec./sec.}$$
$$=4{\cdot}122\text{ ft./sec./sec.}$$

Speed of 30 m.p.h. or 44 ft. per sec. will be lost in

$$\frac{44\text{ ft./sec.}}{4{\cdot}122\text{ ft./sec./sec.}}=10{\cdot}7\text{ sec.}$$

It will thus travel at an average speed of $\frac{44}{2}=22$ ft. per sec. for 10·7 sec. and cover a distance of $22\times10{\cdot}7$ $=235{\cdot}4$ ft.

Example 6. A rail truck runs down an incline of 1 in 80. If the resistance amounts to 14 lb. per ton, find the acceleration and the speed attained, starting from rest, in running 400 yards.

The gravitational force urging the truck downwards and the resistance are proportional to the weight of the truck hence the weight of the truck need not be known. Let it be W lb.

Then the gravitational force along the slope is $\frac{W}{80}$ lb.

This is diminished by a resistance $\frac{14}{2240}$W lb.

Hence the effective accelerating force is $(\frac{1}{80}-\frac{14}{2240})W$ lb.

$$=(0{\cdot}0125-0{\cdot}00625)W \text{ lb.}$$

$$=0{\cdot}00625W$$

$$\text{Hence acceleration}=\frac{0{\cdot}00625W}{W}\times 32{\cdot}2 \text{ ft./sec./sec.}$$

$$=0{\cdot}201 \text{ ft./sec./sec.}$$

If v=maximum speed in feet per second attained in 400 yards or 1200 ft., the time taken is

$$\frac{1200 \text{ ft.}}{\text{average speed}}=\frac{1200 \text{ ft.}}{\frac{1}{2}v \text{ ft./sec.}}=\frac{2400}{v} \text{ sec.}$$

But the time is also $\frac{v \text{ ft./sec.}}{0{\cdot}201 \text{ ft./sec./sec.}}=\frac{v}{0{\cdot}201}$ sec.

$$\text{Hence} \qquad \frac{2400 \text{ ft.}}{v \text{ ft./sec.}}=\frac{v \text{ ft./sec.}}{0{\cdot}201 \text{ ft./sec./sec.}}$$

$$\text{or} \qquad v^2=2400\times 0{\cdot}201 \text{ (ft./sec.)}^2=482{\cdot}4 \text{ (ft./sec.)}^2$$

$$v=22 \text{ ft./sec., nearly.}$$

24. Impulse. The impulse of any constant force during an interval of time is the product of the force and the time. Thus if a force of P lb. acts for t sec. the impulse is

$$P\times t \text{ lb. force sec. units.}$$

If the force is not constant, the same would be true provided P represents the mean or average force, with respect to time, during the interval in which the variable force acts. This could be found by plotting to scale the variable force as ordinates on a time base and finding the mean height which is the area under the graph divided by the length of the base. This mean height would then give, to scale, the mean force.

But we are concerned mainly with constant forces at this stage.

We can now state the second law of motion in another form. Since the force producing a change of motion of a body is proportional to the rate of change of momentum, if the force is constant, the constant rate of change of momentum multiplied by the time will give the total change of momentum, and this will be in the direction of the constant force.

So for a constant force, the force multiplied by the time, or in other words the *impulse*, is proportional to the change of momentum produced in that time. And just as in the previous statement of the second law (Arts. 22 and 23) with a suitable choice of units, the impulse is *equal to* the change of momentum. Thus if a force P lb. is exerted on a body of weight W for t sec. and in consequence its speed rises from v_1 ft. per sec. to v_2 ft. per sec., since

Impulse = gain of momentum

$$\text{P}.t \text{ lb. force sec.} = \frac{\text{W}}{g}v_2 - \frac{\text{W}}{g}v_1 = \frac{\text{W}}{g}(v_2 - v_1) \qquad (1)$$

or generally for any units

✓ $$\text{P}.t = m(v_2 - v_1) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

And again if P is not a constant force, the foregoing statements are true if P stands for the mean force during the t sec. provided the force is always in one direction; in that case the change of v is also in the same direction. We may call such a mean force the time average of the force exerted.

But more generally velocity, momentum, and force have direction as well as magnitude so that the force (and impulse), the change of velocity, and the change of momentum are all to be taken as vector quantities. This will be realised by illustrative examples. But first consider an example of motion in a straight line.

✓ **Example 1.** Solve Example 1 of the preceding article (Art. 23) from considerations of impulse and total gain of momentum.

Let P be the required force in pounds.

$$\text{Impulse} = \text{gain of momentum}$$

$$P \times 10 \text{ sec.} = \frac{1800 \text{ lb. force}}{32{\cdot}2 \text{ ft./sec.}^2} \times 44 \text{ ft./sec.}$$

$$P = \frac{1800 \text{ lb. force} \times 44 \text{ ft./sec.}}{32{\cdot}2 \text{ ft./sec.}^2 \times 10 \text{ sec.}} = 246 \text{ lb. force.}$$

This is, of course, the same as before but it starts from the alternative equation of impulse and a gain of momentum instead of an equation of force and *rate* of gain of momentum (i.e. mass × acceleration). The remainder of the example follows as before.

Example 2. Part of a machine weighing 120 lb. moves at 20 ft. per sec. After an interval of 0·35 sec., its direction of motion has changed through an angle of 45° and it is moving at 24 ft. per sec. Find the change of momentum and the average force acting in the interval.

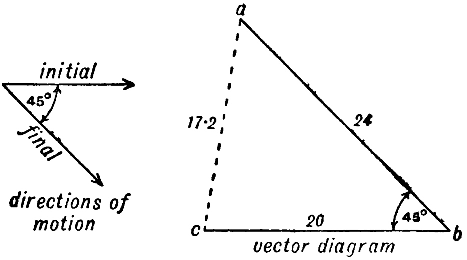

FIG. 16. Vector change of momentum.

We can find the change of momentum by a vector diagram, but as we are concerned with a body of constant mass we may find the change of velocity and hence the change of momentum. Fig. 16 shows on the left the directions of motion at the beginning and end of the period of 0·35 sec.

We draw a vector *ab* in the direction of the final velocity and of such length as to represent 24 ft. per sec. to scale. From this we are to subtract (vectorially) a vector representing 20 ft. per sec. in the initial direction. To do this we *add* such vector in a reversed direction, i.e. from *b* we draw *bc* to represent 20 ft. per sec. to the same scale. Then

$$ac = ab + bc = ab - cb.$$

and *ac* represents the change of velocity. It scales 17·2 ft. per sec. and is in the direction *ac* which is inclined about 100° (measured counterclockwise) to *cb* the initial direction.

The change of momentum is therefore

$$120 \text{ lb.} \times 17{\cdot}2 \text{ ft./sec.} = 2064 \text{ lb.-ft./sec.}$$

or in engineers' units

$$\frac{120}{32{\cdot}2} \times 17{\cdot}2 = 64{\cdot}1 \text{ units,}$$

From the change of momentum, we know the impulse in the 0·35 sec., hence, since

$$\text{Impulse} = \text{change of momentum}$$

$$\text{Force in pounds} \times 0{\cdot}35 \text{ sec.} = 64{\cdot}1$$

$$\text{Force in pounds} = \frac{64{\cdot}1}{0{\cdot}35} = 183 \text{ lb.}$$

Or in foot pound sec. units,

$$\text{Force in poundals} \times 0{\cdot}35 \text{ sec.} = 2064 \text{ lb.-ft./sec.}$$

$$\text{Force in poundals} = \frac{2064}{0{\cdot}35} \text{ lb.-ft./(sec.)}^2$$

$$= 5897 \text{ poundals.}$$

In either case the direction of the average force is in the direction *ac*, namely that of the change of velocity and of momentum.

The change of velocity might have been found easily by resolution instead of by drawing the vector diagram to scale. This alternative method of solution is left as an exercise for the reader.

In the above solution we have taken a total change of velocity, and adopted the statement of the second law of motion from this article. But instead we could have found the average acceleration *f*, viz. 17·2 ft. per sec. divided by 0·35 sec. $= 49{\cdot}1$ ft./(sec.)^2.

Then from the earlier statement of the second law in Art. 22,

$$P=\frac{W}{g}.f=\frac{120 \text{ lb.}\times 49\cdot1 \text{ ft./sec.}^2}{32\cdot2 \text{ ft./sec.}^2}=183 \text{ lb. force.}$$

This illustrates how we can work either from total change of momentum or from the *rate* of change of momentum.

Example 3. A loaded vehicle weighing 12 tons starts from rest drawn by a force P lb. which varies with the time *t* sec. as shown in the following table :

t sec. . . .	0	2	5	8	11	13	16	19	20
P lb. . . .	980	970	920	810	605	500	420	370	360

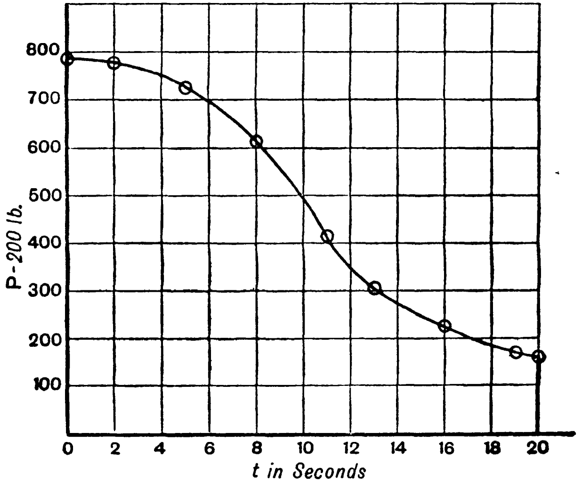

FIG. 17. Impulse of variable force.

If the track and other resistances are constant and equal to 200 lb. force, find the average force effective in accelerating the vehicle during the 20 sec. and the speed at the end of the period.

Subtracting 200 lb. from each of the values of P given in the table, we have the effective force accelerating the vehicle. These are plotted in Fig. 17 and from the graph the effective force at any time can be read, and the average force can be found. Or the area under the curve can be found, e.g. by counting the squares if the graph is plotted to a scale of, say, 200 lb. to 1 in. and 4 sec. to 1 in. (Fig. 17 is reproduced to half the size). The average force is about 476 lb.

The impulse in 20 sec. is therefore

$$476 \text{ lb.} \times 20 \text{ sec.} = 9520 \text{ lb. force-sec.},$$

and this is equal to the gain of momentum. Hence,

$$\frac{W}{g} \times \text{velocity} = 9520$$

$$\frac{12 \times 2240 \text{ lb. force}}{32{\cdot}2 \text{ ft./sec.}^2} \times \text{velocity} = 9520 \text{ lb. force-sec.}$$

$$\text{velocity} = \frac{9520 \text{ lb. force-sec.}}{12 \times 2240 \text{ lb. force}} \times \frac{32{\cdot}2 \text{ ft.}}{\text{sec.}^2}$$

$$= 11{\cdot}4 \text{ ft./sec.}$$

If it were desired to find the distance covered, we could in the same way find the speed at the end of, say, each interval of 4 sec., plot a curve of speeds (as in Art. 7), and from the area under such a graph, or from the *average speed* find the distance travelled in the 20 sec. This will be found an instructive exercise.

25. Third Law of Motion. *To every action there is an equal and opposite reaction.* The word " action " means a force and this is a simple way of stating Newton's Third Law of Motion. We may state this in another way. If any body A exerts a certain force on another body B, then B exerts on A a force of equal magnitude but in the opposite direction. This is equally true if A and B are two parts of the same body which is then in a state of stress.

As applied to bodies in motion, this is only one application of a law which applies also to bodies at rest. Thus in statics we have seen that if a beam or other body exerts a certain downward pressure upon a support, the support exerts an equal upward pressure on the beam, often called a supporting force or a reaction of the support.

But the fact that this is equally true in regard to two bodies in motion is not so widely realised, e.g. that the backward pull of a cart on a horse, or a trailer on a motor vehicle, is equal to the forward pull upon the cart or trailer. Let us look a little more closely at the state of affairs when a motor tractor is pulling a trailer along a level road. If the trailer is moving at constant speed, the forward horizontal pull exerted by the tractor exactly balances the backward horizontal resistance and, so far as a horizontal direction is concerned, we have an illustration of the first law of motion. If, however, the forward pull of the tractor on the trailer exceeds the backward resisting forces upon it, acceleration occurs. But although the forward pull on the trailer exceeds the resistance on the trailer it is still just equal to the backward pull exerted *by the trailer on the tractor.* A pull would still be exerted, however low the resistance might be on a very good track, for it would still result from the inertia of the trailer or its reluctance to accelerate.

Example 1. What force will a man weighing 140 lb. exert on the floor of a lift (1) ascending, (2) descending, with an acceleration of 2 ft. per sec. per sec. in each case.

From the equality of the pressure exerted by the man on the lift and that exerted by the lift on the man, we can find what we require if we can calculate the force exerted on the man by the lift. Let this be P lb. Then from the net accelerating force on the man, his weight and the known acceleration, we can find P. In ascending,

$$\text{Upward accelerating force} = P - 140 \text{ lb.}$$

Then from the proportionality between forces and their resulting acceleration,

$$\frac{(P-140)\text{ lb.}}{140\text{ lb.}} = \frac{2\text{ ft./sec.}^2}{32{\cdot}2\text{ ft./sec.}^2}$$

$$(P-140)\text{ lb.} = \frac{2}{32{\cdot}2} \times 140\text{ lb.} = 8{\cdot}7\text{ lb.}$$

$$P = 148{\cdot}7\text{ lb.}$$

In descending,

$$\frac{(140-\text{P})\text{ lb.}}{140\text{ lb.}}=\frac{2\text{ ft./sec.}^2}{32{\cdot}2\text{ ft./sec.}^2}$$

$$(140-\text{P})\text{ lb.}=8{\cdot}7\text{ lb.}$$

$$\text{P}=140-8{\cdot}7=131{\cdot}3\text{ lb.}$$

Example 2. Two weights W_1 lb. and W_2 lb. are connected by a light inextensible string passing over a pulley, having only negligible mass and bearing friction, as shown in Fig. 18, W_1 being greater than W_2. Find the acceleration and the tension in the string. (This essentially represents Atwood's machine.)

The frictionless pulley has the same effect as if the string were sliding over it without friction and the tension will be the same throughout the string. Let it be P lb. There are two unknown quantities, viz. the acceleration f and the tension P. But by considering separately the acceleration of the weights W_1 and W_2, we shall get two simultaneous equations from which we can find P and f.

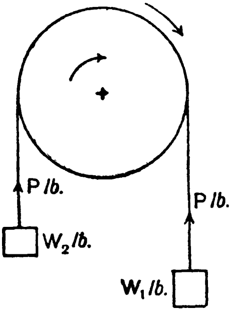

Fig. 18. Acceleration problem.

Considering first the weight W_1, the proportionality of acceleration to the force, for a downward acceleration of f ft. per sec.² (as in (1), Art. 20) tells us that

$$\frac{W_1-P}{W_1}=\frac{f}{g}\quad\text{or}\quad W_1-P=\frac{W_1}{g}f\text{ lb.}\qquad .\quad(1)$$

Similarly for an upward acceleration of W_2

$$\frac{P-W_2}{W_2}=\frac{f}{g}\quad\text{or}\quad P-W_2=\frac{W_2}{g}f\text{ lb.}\qquad .\ .\quad(2)$$

adding equations (1) and (2)

$$W_1-W_2=\frac{W_1+W_2}{g}f$$

or

$$f=\frac{W_1-W_2}{W_1+W_2}g.$$

And this would be equally true if W_1 and W_2 were in, say, grammes and g in centimetres per second per second.

And from equation (1),

$$P = W_1\left(1 - \frac{f}{g}\right) \text{ lb.} = \frac{2W_1W_2}{W_1 + W_2} \text{ lb.}$$

✓ **Example 3.** A sliding block B and an added weight (Fig. 19) can slide over a horizontal track A and together weigh 10 lb. They are drawn along by a total weight of 4 lb. hanging vertically by a cord, passing over a pulley of negligible weight and friction, which gives a horizontal pull on the sliding block. If the coefficient of friction between the sliding block and the track A is 0·2, find the acceleration of the slider and the tension in the cord.

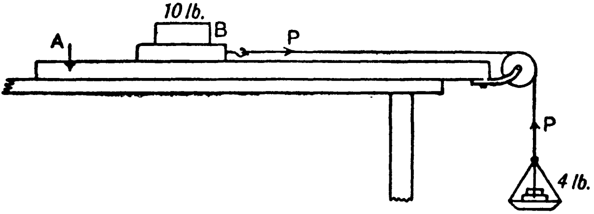

FIG. 19. Acceleration and friction problem.

Here again we have two unknown quantities, viz. the tension of the cord P lb. and the acceleration f.

Considering the descending weight of 4 lb., from the proportionality of acceleration to force

$$\frac{(4 - P) \text{ lb.}}{4 \text{ lb.}} = \frac{f}{32{\cdot}2 \text{ ft./sec.}}$$

$$\text{or} \quad (4 - P) \text{ lb.} = 4 \times \frac{f}{32{\cdot}2} \frac{\text{lb.}}{\text{ft./sec.}^2} \quad . \quad . \quad (1)$$

and considering the 10 lb. sliding body, the net force in the direction of motion is P minus the frictional drag.

The friction is $0{\cdot}2\times10=2$ lb. force. Hence from the proportionality of acceleration and force

$$\frac{(\mathrm{P}-2)\text{ lb.}}{10\text{ lb.}}=\frac{f}{32{\cdot}2\text{ ft./sec.}^2}$$

$$\text{or}\quad(\mathrm{P}-2)\text{ lb.}=10\times\frac{f}{32{\cdot}2}\,\frac{\text{lb.}}{\text{ft./sec.}^2}\quad .\ .\quad(2)$$

Adding equation (1) and (2) to eliminate P,

$$(4-2)\text{ lb.}=(4+10)\frac{f}{32{\cdot}2}\,\frac{\text{lb.}}{\text{ft./sec.}^2}$$

hence $f=\frac{2}{14}\times32{\cdot}2\text{ ft./sec.}^2=4{\cdot}6\text{ ft./sec.}^2$,

and substituting this in equation (1),

$$(4-\mathrm{P})\text{ lb.}=4\text{ lb.}\times\frac{4{\cdot}6\text{ ft./sec.}^2}{32{\cdot}2\text{ ft./sec.}^2}=0{\cdot}57\text{ lb.}$$

$$\mathrm{P}=4\text{ lb.}-0{\cdot}57\text{ lb.}=3{\cdot}43\text{ lb.}$$

26. Conservation of Momentum. An important consequence of the third law (or an alternative statement of it) is that the total momentum of two bodies is unaltered by any action between them. For the force exerted by A on B is of the same magnitude as that exerted in the opposite direction by B on A. Consequently the impulse (i.e. force × time) given in any time by A to B is the same as that given by B to A in the opposite direction. Since impulse equals the change of momentum (from Art. 24) it follows that if B gains any momentum, A loses an equal amount and the total change for the two bodies jointly is zero. And this is true for any and every direction.

This is expressed in the statement that the momentum in any direction, of two bodies free from any external force is constant, And this conservation of momentum extends to any system of bodies isolated from external force and is in no way limited to two bodies.

As an example of the principle, if two balls of equal diameter (but not necessarily equal mass) moving horizontally on a smooth surface collide, they may subsequently move in the same or in different directions, but the total momentum in any direction will be unaffected by the collision.

And when a projectile is fired from a gun, the equal and opposite forces exerted by the gun on the projectile and the projectile on the gun have equal impulse and the change of momentum of the projectile due to firing the charge is equal and opposite to that of the recoiling heavier gun which moves backward more slowly. The momentum of recoil is transmitted to the earth and so ultimately is that of the shot, the net momentum given to the earth being zero. (The earth here includes the atmosphere.)

It is worth noting that the conservation of momentum in any action is unaffected by friction. A frictional drag, acting on a body tangential to its direction of motion, is a force by means of which momentum may be exchanged between a moving body and the earth, just as by a direct thrust or pull.

And in any interchange of momentum, one body of a pair may be the earth. Its change of momentum due to force exerted by another body may be quite imperceptibly small but the principle is still true.

Example 1. A bullet weighing 1 oz. is fired into a suspended block of wood which weighs 25 lb. If the block acquires a speed of 6 ft. per sec., what was the velocity of the bullet?

The solution of the problem depends upon the fact that since the bullet becomes embedded in the wood, the total mass of 25 lb. and 1 oz. moving at 6 ft. per sec. has the same total momentum as the bullet alone had before the impact. We can equate the momenta in any units, but pound mass × feet per second will be the simplest. Let v be the speed of the bullet. Then,

$$\tfrac{1}{16}\text{ lb.}\times v=25\tfrac{1}{16}\text{lb.}\times 6\text{ ft./sec.}$$

$$v=\frac{25\cdot0625}{0\cdot0625}\times 6\text{ ft./sec.}=401\times 6\text{ ft./sec.}=2406\text{ ft./sec.}$$

Example 2. A gun weighing 60,000 lb. fires a projectile weighing 800 lb. with a velocity of 1000 ft. per sec.

With what velocity will the gun recoil? If the time average of the force overcoming the recoil is 120,000 lb., how far will the gun travel and how long will it take?

From the equality of the impulses resulting from firing the gun the momentum of the gun, and that of the projectile are equal. If the momentum be stated in pounds × feet per second and if v be the velocity of the gun in feet per second,

$$60{,}000 \text{ lb.} \times v = 800 \text{ lb.} \times 1000 \text{ ft./sec.}$$
$$v = 13{\cdot}33 \text{ ft./sec.}$$

And since the impulse of the retarding force must equal the loss of momentum by the gun,

$$\text{Retarding force in pounds} \times \text{time in seconds} = \frac{60{,}000 \text{ lb. force}}{32{\cdot}2 \text{ ft./sec.}^2} \times 13{\cdot}33 \text{ ft./sec.}$$

$$120{,}000 \text{ lb. force} \times \text{time} = \frac{60{,}000 \text{ lb. force} \times 13{\cdot}33 \text{ ft./sec.}}{32{\cdot}2 \text{ ft./sec.}^2}$$

$$\text{time} = \frac{60{,}000 \text{ lb. force} \times 13{\cdot}33 \text{ ft./sec.}}{120{,}000 \text{ lb. force} \times 32{\cdot}2 \text{ ft./sec.}^2}$$
$$= 0{\cdot}207 \text{ sec.}$$

$$\text{Average speed} = \frac{13{\cdot}33}{2} \text{ ft./sec.} = 6{\cdot}67 \text{ ft./sec.}$$

$$\text{Distance moved} = 6{\cdot}67 \text{ ft./sec.} \times 0{\cdot}207 \text{ sec.}$$
$$= 1{\cdot}38 \text{ ft.}$$

27. Rotation about an Axis. In this chapter we have been concerned with linear motion, its laws and the relation of force to mass and acceleration. Very similar laws hold in regard to rotary motion about an axis, torque or turning moment, rotary inertia and angular acceleration. We cannot pursue these far at this stage but we can easily establish the relation between torque and the rate of increase of angular velocity.

Suppose that a small body or a small part of a larger body is rotating, or is free to rotate, about an axis. Let W be its weight in pounds and r be its distance from the axis of rotation in feet, and let it have a linear rate of increase of speed of f ft. per sec. per sec. Then the force tangential to its circular path to accelerate it will be (Art. 20)

$$P = W.\frac{f}{g} \text{ lb. force} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$= \frac{W}{g} ra \text{ lb. force} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where a is the angular acceleration in radians per second per second (see (7), Art. 11). The torque or turning moment T on the shaft necessary to exert the force P on the body will be

$$T = P \text{ lb. force} \times r \text{ ft.} = Pr \text{ lb.-ft.} \quad . \quad . \quad (3)$$

or

$$T = \frac{W}{g} ra \times r = \frac{W}{g} r^2 a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The body may be a thin rim of a wheel, all the mass of which is situated at practically the same distance r from the axis. But in general the distance of the parts of a body from the axis of rotation will differ so that for several parts, of weights W_1, W_2, W_3, etc., at radii r_1, r_2, r_3, etc., the total torque required to give the complete body an angular acceleration a would be

$$T = \left(\frac{W_1}{g} r_1^2 + \frac{W_2}{g} r_2^2 + \frac{W_3}{g} r_3^2 + \ldots, \text{ etc.}\right) a$$

or for the total weight W,

$$T = \frac{W}{g} k^2 a \text{ lb.-ft.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where k^2 is the mean of the squares of the distances in feet of a large number of equal parts from the axis of rotation, or a " weighted " mean if the parts are not all of equal weight.

Such a radius k is called the radius of gyration, and $\frac{W}{g} k^2$ is the measure of the rotary inertia about the particular axis. The radius k is that at which the whole mass would, if concentrated, have the same rotary inertia, or reluctance to take up rotation, as the actual body has. The rotary inertia is usually called the moment of inertia and is denoted by I.

In the pound-foot-second system, it will have units of pounds × feet^2 or $I = m.k^2$ = mass in pounds × (k in feet)^2.

That the acceleration is constant under the exertion of a constant torque, and that with different torques the angular accelerations are proportional to the torques may readily be shown on the small experimental flywheels to be found in Mechanics laboratories.

The relation (5) for rotation corresponds to that for Newton's Second Law for linear motion in (2), Art. 20. The statement concerning angular motion corresponding to Newton's First Law, is that a body spins about any axis with constant angular velocity except in so far as it is compelled by external torque to alter that rotation.

And corresponding to the Third Law of motion we have the fact that torque (like force) has a dual action so that if a body A exerts a torque on another body B, then B exerts an equal and opposite torque on A.

Example. A flywheel weighs 10 tons and has a radius of gyration of 5 ft. What effective torque will be required to raise its speed from 60 r.p.m. to 80 r.p.m. in 40 sec.? And if it then comes to rest under the exertion of a constant frictional torque of 500 lb.-ft. how many revolutions will it make before stopping and how much work will it do against friction?

$$\text{Angular acceleration} = \frac{(80-60)\text{ r.p.m.}}{40\text{ sec.}} = 0{\cdot}5\text{ r.p.m./ sec.}$$

$$= 0{\cdot}00833\text{ revs./sec./sec.,}$$

or in radians per sec. per sec.

$$a = 2\pi \times 0{\cdot}00833\text{ radians/sec.}^2$$

$$= 0{\cdot}05236\text{ radians/sec./sec.}$$

Torque required

$$= \frac{10 \times 2240\text{ lb. force}}{32{\cdot}2\text{ ft./sec.}^2} \times 5^2\text{ ft.}^2 \times 0{\cdot}05236\text{ radian/sec.}^2$$

$$= 910\text{ lb.-ft.}$$

In stopping,

$$\text{Angular deceleration } a = \frac{\text{torque}}{\text{rotary inertia}}$$

$$= \frac{500 \text{ lb.-ft.} \times 32{\cdot}2 \text{ ft./sec.}^2}{10 \times 2240 \text{ lb. force} \times 5^2 \text{ ft.}^2}$$

$$= 0{\cdot}0288 \text{ radians/sec./sec.}$$

Initial angular velocity at 80 r.p.m.

$$= \frac{80 \times 2\pi}{60} = 8{\cdot}38 \text{ radians/sec.}$$

Time to lose this angular velocity

$$= \frac{8{\cdot}38 \text{ radians/sec.}}{0{\cdot}0288 \text{ radians/sec.}^2} = 291 \text{ sec.,}$$

or 4 min. and 51 sec.

This result could also be calculated from the accelerating torque found above since the time taken is inversely proportional to the torque and proportional to the total reduction of speed. Hence since a torque of 910 lb.-ft. changes the speed by 20 r.p.m. in 40 sec., the time to change the speed by 80 r.p.m. (to zero) by a torque of 500 lb.-ft. will be

$$40 \text{ sec.} \times \tfrac{910}{500} \times \tfrac{80}{20} = 291 \text{ sec.}$$

Total revolutions in coming to rest at an average speed of 40 r.p.m.

$$= 40 \text{ r.p.m.} \times \tfrac{291}{60} \text{min.} = 194 \text{ revs.}$$

$$\text{Work done} = 500 \text{ lb.-ft.} \times 2\pi \times 194 = 609{,}500 \text{ ft.-lb.}$$

28. Centrifugal Force. The previous article dealt with the forces (and therefore torques) required to accelerate the rotation of a body about an axis. But when a body is rotating about an axis whether at a changing or at a *constant* angular speed any part of it is constantly changing its direction of motion, that is, it does not continue to move in a straight line. Therefore, although its *speed* may remain constant its *velocity* changes, for it changes its direction whether or not it changes its magnitude. Consequently the body must be acted upon by a force in order to make it change its direction of motion; otherwise the first law of motion tells us it would move in a straight line with constant velocity. The resultant force would be the same for, say, a mass of 1 oz. whether it were an ounce of metal on the rim of a heavy flywheel or whether it were an isolated piece of metal weighing 1 oz. whirled in a circle at the end of a string provided the radius from the axis and the speed were the same. But to simplify the conditions it is better at this stage to think of the small isolated body on which the force constraining it to turn in a circular path is applied by a thin string. For obviously the only force exerted upon it by the string is a pull along the string.

And the only forces which can act on the body are (1) the pull of the string and (2) the gravitational pull of the earth (equal to the weight of the body).

We have already investigated in Art. 13 the motion of a point moving in a circular path of radius r ft. at a linear velocity v ft. per sec. corresponding to an angular velocity ω radians per sec. about the centre of the circle so that $\omega=v/r$. And we found that the point had an acceleration

$$\frac{v^2}{r} \quad \text{or} \quad \omega^2 r \text{ ft./sec./sec.}$$

directed from the moving point, radially inward towards the centre of the circle. Or from A towards O in Fig. 14. Applying the principle embodied in the second law of motion to this we know that a body thus moving in a circle must have exerted upon it a definite force directed also radially inward towards the centre of the circle. This is called the centripetal force acting on the body moving in a circle. And (2) of Art. 20, or (2) of Art. 23, tell us that this centripetal force is

$$\mathrm{P}=\frac{\mathrm{W}}{g}\times f=\frac{\mathrm{W}}{g}\,\frac{v^2}{r} \quad \text{or} \quad \frac{\mathrm{W}}{g}\omega^2 r \text{ lb.} \quad . \quad . \quad . \quad (1)$$

acting radially inwards.

Reverting now to the small body at the end of a string, let us call it a stone and let it be moving in a circular path of radius r ft. as shown at the left side of Fig. 20, with constant speed v ft. per sec. It is attached by a string to a point B vertically above the centre of motion O. Then if W lb. is the weight of the stone and T lb. is the tension of the string the resultant of T and W must be a force $\frac{W}{g}\frac{v^2}{r}$ lb. from A towards O. Or if abc is a vector diagram of the forces exerted on the stone,

$$ab+bc=ac$$

and the force ac is the resultant force *on the stone.* Note that this is not a case of equilibrium; ac is a

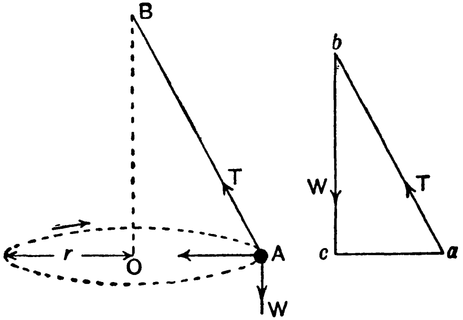

FIG. 20. Resultant force on body in circular motion.

resultant force. The body is accelerated motion, and therefore has a resultant force upon it. (Problems on circular motion are sometimes solved by adding to the forces actually exerted on the moving body, a *purely fictitious* or imaginary outward radial force equal and opposite to the actual inward force and then regarding the body as in equilibrium. While this will give the magnitude of the forces involved, it obscures the facts of the situation and is very liable to give the impression that the body has this outward force exerted upon it which it has *not.* So we do not recommend this artifice.)

The arrangement in Fig. 20 is called a conical pendulum, neither of the two forces T and W does any work on the body. For they act at right angles to the direction of motion.

Centrifugal force is a radially outward force exerted *by* a body moving in a circular path *upon* whatever constrains it to follow that path. In other words it is the reaction of the centripetal force. From the third law of motion we know that forces occur in pairs in this way. In this matter of centrifugal force misunderstanding is very common. Persons moving fast on foot, on cycle, or in a motor car and failing to turn as sharp a corner as they have attempted to do are liable to think they are acted upon by an outward force. Actually, in accordance with the first law, on account of inertia, they continue in a straight line except in so far as there is force to deviate them from it. And if the force available is insufficient they turn a less sharp corner than they intended or hoped to do. So far as the intended path is concerned they side-slip or skid, not due to any outward force but for lack of an inward force to change their direction of motion. It is instructive to watch drops of water flying tangentially (*not* radially) off the rim of a wet wheel when inward forces fail to hold them.

The centrifugal force exerted *by* a body on whatever constrains its motion, though not a force acting *on* the body, is of course important. In Fig. 20 the centrifugal force exerted by the stone on the string contributes to the tension in the string: its magnitude, $\frac{W}{g}\frac{v^2}{r}$, increases in proportion to the square of the speed, and when the speed is very great and the string is nearly horizontal the tension is nearly wholly due to centrifugal force exerted by the stone.

Centrifugal force is exerted by a train on the rails when running on a curved track. If the rails were level they might be moved outwards by the centrifugal force exerted sideways upon them. But the outer rail is raised and thus the thrust on the rails of the centrifugal force and the weight of the train may have a resultant normal or perpendicular to the face of the rails. And the reaction of the rails on the train supplies the inward component which is the centripetal force acting *on the* *train*. Similarly with a banked cycle or motor track or curved road; the inward component of the reaction of the banked track supplies the centripetal force necessary to the moving body if it is to turn, i.e. to alter its direction. And if the track is banked the reaction of the track need not be so far from the normal as to exceed the angle of friction and cause slipping sideways. Except at one particular speed there will not be an exactly normal reaction, but the deviation from the normal will be limited. We have considered the centripetal force exerted *on* and the centrifugal force exerted *by* a small isolated body, the speed of which can be treated as that of a point. But in bodies of appreciable size, the centripetal and centrifugal forces of magnitude

$$\frac{W}{g}\frac{v^2}{r} \quad \text{or} \quad \frac{W}{g}\omega^2 r,$$

the speed v and the distance r from the axis of motion are to be taken as those at the centre of gravity of the body. Thus for a perfect wheel with the centre of gravity in its axis, the resultant centrifugal force exerted on its shaft, due to rotation about the axis, will be zero.

Example 1. A block of cast iron, 4 in. × 3 in. × 4 in., weighs 12·5 lb. and is fastened to a disc with its c.g. 3 ft. from the axis, about which the disc makes 600 r.p.m. What is the force tending to fracture the fastening?

The speed of its c.g. is $\frac{2\pi \times 3 \times 600}{60} = 60\pi = 188{\cdot}6$ ft./sec.

Hence its pull on the fastening or the centrifugal force it exerts is

$$\frac{W}{g}\frac{v^2}{r} = \frac{12{\cdot}5 \text{ lb.} \times 188{\cdot}6^2 \text{ (ft./sec.)}^2}{32{\cdot}2 \text{ ft./sec.}^2 \times 3 \text{ ft.}}$$

$$= \frac{12{\cdot}5 \times 188{\cdot}6 \times 188{\cdot}6}{32{\cdot}2 \times 3} \text{ lb. force}$$

$$= 4600 \text{ lb. force.}$$

Example 2. A flywheel weighs 600 lb. and its c.g. is 0·1 in. from the axis of rotation. What lateral pull will the wheel exert on the shaft at 500 r.p.m.?

The radius to the c.g. is 0·1 in. $=\frac{1}{120}$ ft.

The angular velocity $=\omega=\dfrac{2\pi\times500}{60}=52{\cdot}4$ radians/sec

$$\text{Pull on shaft}=\frac{W}{g}\omega^2r=\frac{600\text{ lb. force}}{32{\cdot}2\text{ ft./sec.}^2}\times\frac{52{\cdot}4^2\text{ radians}^2}{\text{sec.}^2}\times\frac{1\text{ ft.}}{120}$$

$$=\frac{600\times52{\cdot}4\times52{\cdot}4}{32{\cdot}2\times120}=426\text{ lb. force.}$$

29. Balancing. All parts of a body rotating about an axis exert centrifugal force, and if the body is fastened to a shaft rotating in bearings, the centrifugal forces of the various parts will exert a resultant lateral pull on the shaft unless they are so arranged that together they give a resultant pull of zero. Even then they might reduce to a couple tending to make the whole rotating mass wobble and so bring forces to bear on the shaft. When the whole of the rotating mass is so arranged that there is no resultant centrifugal force or couple on the shaft it is said to be "balanced." The subject of balancing is a large one and at this stage we can only touch it and we shall greatly simplify the work by limiting our consideration to forces in one plane perpendicular to the axis of rotation. Suppose, for example, we have a circular disc such as the face plate of a lathe to which we can attach pieces of metal. When the disc rotates about its axis, each piece will exert a centrifugal force; and we shall examine how we can arrange that the total pull on the spindle is zero, or if it is not, what additional piece or pieces of metal will make the total resultant centrifugal force zero, i.e. will balance the other pieces. So long as we limit ourselves to one plane of rotation we shall have balanced the pieces of metal if we so arrange them that the c.g. of the whole is in the axis of rotation. But the principle of balancing can best be illustrated by simple examples.

Example 1. A casting weighing 56 lb. is bolted to the face plate of a lathe with its c.g. 18 in. from the axis of rotation. What mass placed diametrically opposite to it at a distance of 15 in. from the axis will balance this casting?

Firstly, we notice that since the centrifugal forces are $\frac{W}{g}\omega^2 r$, it follows that if they are equal and opposite at one speed, they will be so at another, so we do not need to be told the speed of running. The angular velocity can be denoted by ω radians per sec.

Centrifugal force exerted by the casting is

$$\frac{W}{g}\omega^2 r = \frac{56 \text{ lb. force}}{32{\cdot}2 \text{ ft./sec.}^2} \times \omega^2 \text{ (radians/sec.)}^2 \times \frac{18}{12} \text{ ft.}$$

$$= \frac{56}{32{\cdot}2} \times 1{\cdot}5\omega^2 \text{ lb. force.}$$

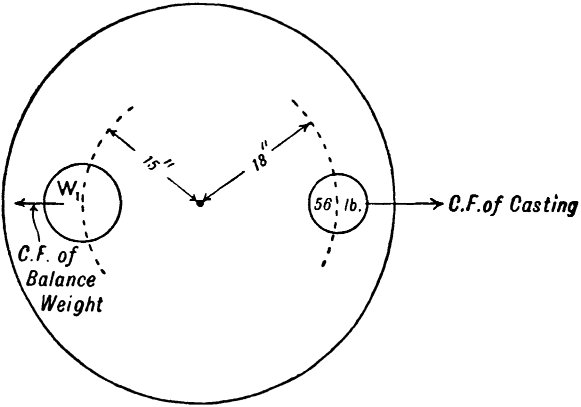

FIG. 21.—Balancing centrifugal forces.

Let W_1 be the weight of the mass necessary to balance it at 15-in. radius (see Fig. 21). Then this mass exerts an opposite force

$$\text{c.f.} = \frac{W_1 \text{ lb. force}}{32{\cdot}2 \text{ ft./sec.}^2} \times \omega^2 \text{(radians/sec.)}^2 \times \frac{15}{12} \text{ ft.}$$

$$= \frac{W_1}{32{\cdot}2} \times 1{\cdot}25\omega^2 \text{ lb. force.}$$

This is to be equal to the c.f. of the casting, hence

$$\frac{W_1}{32\cdot2}\times1\cdot25\omega^2=\frac{56}{32\cdot2}\times1\cdot5\omega^2$$

$$W_1=56\times\frac{1\cdot5}{1\cdot25}\text{ lb.}=67\cdot2\text{ lb.}$$

It will be evident that ω and 32·2 are not necessary in equating the effects of the two centrifugal forces. If the c.f. of a mass weighing W_1 lb. at radius r_1 ft. is to equal the c.f. of another mass weighing W_2 lb. at r_2 ft.

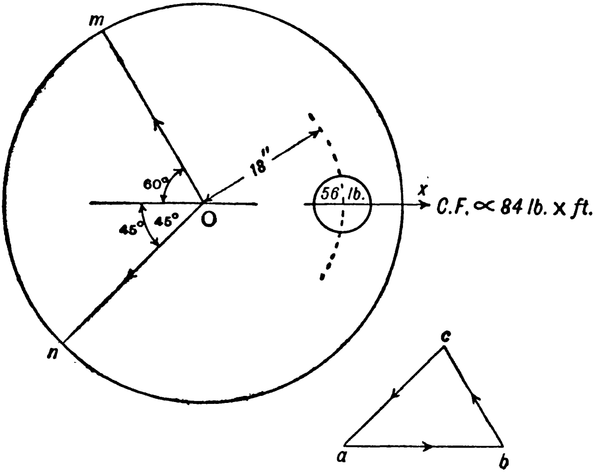

FIG. 22.—Balancing one mass by two.

$$\frac{W_1}{g}\omega^2r_1=\frac{W_2}{g}\omega^2r_2$$

or
$$W_1r_1=W_2r_2$$

The products such as W lb. × r ft. serve to determine W_1 at a given radius.

Example 2. How could the casting in Example 1 be balanced by two masses in radial positions indicated by Om and On in Fig. 22?

This is like a statical problem and may be expressed as what two products Wr along Om and On will balance a product W.r=56 ×1·5 =84 lb.-ft. along Ox ?

It is easily solved by drawing the triangular vector diagram. Thus draw *ab* parallel to Ox and representing 84 lb.-ft. to scale. Then draw *bc* and *ca* parallel to Om and On respectively; then $bc+ca=ba$ which is equal and opposite to *ab*. *bc* scales 61·5 lb.-ft. and *ca* 75·3 lb.-ft. Any masses and radii giving these products will satisfy the conditions. And we are free to chose, say, radii if they are not given in the problem. Suppose the mass on radius Om is to have its c.g. 18 in. (=1·5 ft.) from O and that on On is to have its c.g. 1·25 ft. from O. Then

$$\text{mass in O}m\times 1{\cdot}5\text{ ft.}=61{\cdot}5\text{ lb.-ft.}$$

$$\text{mass}=\frac{61{\cdot}5}{1{\cdot}5}=41\text{ lb.}$$

$$\text{mass in O}n\times 1{\cdot}25\text{ ft.}=75{\cdot}3\text{ lb.-ft.}$$

$$\text{mass}=\frac{75{\cdot}3}{1{\cdot}25}=60{\cdot}2\text{ lb.}$$

Similarly by the use of a vector polygon we could find balance masses in given directions for any number of otherwise unbalanced masses, for the centrifugal forces exerted by several, all passing through the axis would all reduce to a single resultant, and the vector diagram would be an open polygon to be closed by one or by two sides. It could only be closed by one single side provided this were in the same straight line as the resultant centrifugal force exerted by the masses which have to be balanced.

Summary of Chapter II

First Law of Motion. A body moves in a straight line with constant speed unless compelled by force to do otherwise. It therefore possesses inertia or reluctance to change and the measure of this is mass. Weight is the gravitational attraction of the earth.

The acceleration of a body is proportional to the force causing it and in particular,

$$\frac{f}{g}=\frac{P}{W}, \text{ or } P=\frac{W}{g}.f$$

The momentum of a body is measured by the product of its mass and its velocity.

Second Law of Motion. The rate of change (gain) of momentum of a body measures the magnitude of the force exerted upon it and is in the same direction, or $P \propto mf$. And with suitable units $P=mf$. If m is in pounds and f in feet per second per second, P is in poundals, but if m is taken as W/g, P is in pounds force.

The impulse of a constant force is $P \times t$ and is equal to the change of momentum in the time t.

Third Law of Motion. To every action there is an equal and opposite reaction. Or forces occur in pairs. If A exerts a force on B, then B exerts an equal and opposite force on A. (It is important not to confuse a force exerted *by* a body with a force exerted *on* it.) If follows that the total momentum of A and B is not altered by their mutual actions.

Corresponding laws hold in regard to rotary motion and in particular,

$$T=\frac{W}{g}k^2a$$

A body moving in a circle at uniform speed has a force

$$\frac{W}{g}\omega^2r \quad \text{or} \quad \frac{Wv^2}{gr}$$

exerted upon it directed to the centre of the circle (centripetal force) and the body exerts an equal force radially outward on whatever holds it to its circular path (centrifugal force). A body is balanced for rotation about an axis when the centrifugal forces exerted by its parts have resultant zero, and a zero moment about all axes. Resultants and equilibrants can be found for centrifugal forces by the rules applicable to statics.

EXAMPLES II

1. A car weighing 1 ton is moving on level ground at 8 m.p.h. and after 6 sec. its speed has increased to 20 m.p.h. What average effective force in the direction of motion was exerted on the car during that time ?

2. What constant retarding force will bring to rest in 15 sec. a train weighing 500 tons and moving at 60 m.p.h. on a level track ?

3. A truck weighing 15 cwt. is pulled on a level line by a force of 60 lb. in excess of the frictional resistance. Find the acceleration of the truck and the distance it will move in ½ min. What is the horse-power if frictional resistance amounts to 14 lb. ?

4. What average pull will be required on a level track to give a train of 400 tons a speed of 40 m.p.h. from rest in 2 min. if frictional resistance is 12 lb. per ton ? What would the horse-power be at the end of the 2 min ?

5. A tram car weighs 5 tons and attains from rest a speed of 12 m.p.h. in 15 sec. up an incline of 1 in 30. If the frictional resistances are 15 lb. per ton find the horse-power necessary at the end of the period.

6. A body weighing 50 lb. is moving at 30 ft. per sec. due north and after 1·5 sec. it is moving south-west at 40 ft. per sec. Find the average force which has been exerted upon it during this time.

7. A 500-lb. mass is moving at a speed of 20 ft. per sec. After ½ min. it is moving at 30 ft. per sec. in a direction at right angles to its former path. Find the mean acceleration in this period and the average force required to produce this change of velocity.

8. A train starting from rest is drawn along a level track by a force of P lb. which after *t* sec. has the values shown in the following table :

t sec. . .	0	11	20	34	45	55	60
P lb. . .	40,100	39,150	37,600	35,300	33,700	32,100	30,000

If the train weighs 300 tons and has frictional resistance of 12 lb. per ton, what is the time-average of the force P exerted in the 60 sec. and what speed will the train have attained in that time ?

9. A wheel is making 200 r.p.m. and after 20 sec. its speed has fallen to 150 r.p.m. If the angular retardation is constant, how many revolutions will the wheel make in coming to rest from 200 r.p.m. ?

10. A wheel weighs 10,000 lb. and has a radius of gyration of 4 ft. With a driving torque of 1000 lb.-ft. and a resisting torque of 100 lb.-ft. how long will the wheel take to increase in speed from 20 r.p.m. to 250 r.p.m. ?

11. A flywheel weighs 2 tons and has a radius of gyration of 3 ft. What resisting torque will be required to bring it to rest in 1 min. from a speed of 120 r.p.m. and how much work will it do against this resistance ?

12. The blade of a steam turbine running at 15,000 r.p.m. weighs 0·03 lb. and its c.g. is at 10 in. from the axis of rotation. Find the centrifugal pull exerted by the blade on the rotor to which it is attached.

13. If a flywheel weighing 10 tons has its c.g. eccentric by 0·02 in. from the axis of rotation what pull will result on the shaft at 240 r.p.m. due to the eccentricity ? What weight at 2 ft. radius would correct the eccentricity ?

14. A casting weighing 56 lb. is bolted to the face plate of a lathe with its c.g. 8 in. from the axis of rotation. What pull will it exert on its fastenings at 300 r.p.m. and what weight placed diametrically opposite at a radius of 10 in. will just balance it ?

15. The casting in No. 14 is to be balanced by two weights, the first placed at an angle of 135° and the second at 240° (both measured in the same direction) from the c.g. of the casting. Find the weights required if the first has its c.g. 12 in. and the second 15 in. from the axis.

CHAPTER III

ENERGY

30. Energy. In a previous course we have seen that energy can take various forms. A body may, by being raised vertically against the force of gravity, be capable of doing work in returning to its former position and thereby possess, when elevated, potential energy or energy of position. And a spring, when elastically strained, possesses elastic strain energy: it can do work in transferring its energy to another body. And it is a matter of common experience that a body in motion overcomes resistance in being brought to rest and so does work. Also that to accelerate it and give it speed requires force exerted through a distance: in the previous chapter we have examined what force is required to accelerate a body; it is proportional to its mass and the acceleration. Thus a body in motion possesses a definite amount of *kinetic energy* or energy of motion. It is our next purpose to examine this quantitatively and find its relation to other forms of energy.

31. Kinetic Energy. If a force is exerted upon a body and there is no opposing resistance except the inertia of the body, the whole of the work done is a measure of the energy from some external source which becomes the kinetic energy of the body. Suppose a constant force P lb. is thus exerted through a distance s ft. giving the body a speed v ft./sec. in t sec., the acceleration being f ft./sec./sec.

Then the work done representing energy transferred to the body is

$$P \times s \text{ ft.-lb.}$$

And since $P=\frac{W}{g}.f$ (Art. 20)

and $s=\frac{1}{2}vt$, i.e. average speed in ft./sec. × time in seconds,

the kinetic energy is

$$P\times s \text{ ft.-lb.}=\frac{W}{g}.f\times\frac{v.t}{2}$$

$$=\frac{W}{g}\frac{v}{t}\times\frac{vt}{2}=\frac{1}{2}\frac{W}{g}v^2 \quad . \quad . \quad . \quad . \quad (1)$$

and the units are $\frac{\text{lb.-force}}{\text{ft./sec.}^2}\times\frac{(\text{ft./sec.})^2}{1}$ or ft.-lb.

And if the initial speed were u instead of zero, we should have kinetic energy

$$P\times s=P\times\text{mean velocity}\times\text{time.}$$

$$=P\times\frac{u+v}{2}\times t$$

$$=\text{rate of gain of momentum}\times\frac{u+v}{2}\times t$$

$$=\frac{W}{g}\frac{(v-u)}{t}\frac{u+v}{2}\times t=\frac{1}{2}\frac{W}{g}(v^2-u^2) \quad . \quad . \quad (2)$$

which is the final minus the initial kinetic energy.

We have here assumed a constant force P and consequently a constant acceleration, but the relations (1) and (2) are equally true when P is the mean value (space-average) of the varying force. That is to say, the gain of kinetic energy of a body, in foot-pounds, is equal to the work done in foot-pounds in transferring energy from some other body, such as an elastically strained spring, to the body set in motion or given increased speed.

It is worth noting that while v is a vector quantity and has direction, neither v^2 nor $\frac{1}{2}\frac{W}{g}v^2$ are vector quantities. They have no direction.

Also, since we may write m in place of $\frac{W}{g}$ for the mass of a body, we may write the kinetic energy as $\frac{1}{2}mv^2$, a convenient form for any units. When a body such as a train or a motor-car has considerable kinetic energy due to travelling at a high speed this is generally spent, when coming to rest, in overcoming tractive resistance, often increased by brakes. The energy is then lost as heat.

Problems on force and motion can often be solved by considering the kinetic energy and the work done by propelling forces, or by methods given in the previous chapter. The alternative methods are illustrated in Examples 1, 2, and 3 following.

✓**Example 1.** A motor vehicle weighs 1 ton and is running on a level road at 30 m.p.h. What is its kinetic energy in foot-pounds? If the resistance to motion is 30 lb. per ton, how far will it run, without driving force or brakes before coming to rest?

30 m.p.h. being 44 ft./sec., the kinetic energy

$$\frac{1}{2}\frac{Wv^2}{g}=\frac{2{,}240 \text{ lb. force}\times 44^2(\text{ft./sec.})^2}{2\times 32{\cdot}2 \text{ ft./sec.}^2}$$

$$=\frac{2{,}240\times 44\times 44 \text{ ft.-lb.}}{2\times 32{\cdot}2}=67{,}340 \text{ ft.-lb.}$$

In expending this energy,

$$\text{work done in resistance}=30 \text{ lb.}\times\text{distance run}$$

$$=67{,}340 \text{ ft.-lb.}$$

$$\text{distance}=\frac{67{,}340 \text{ ft.-lb.}}{30 \text{ lb.}}=2{,}244 \text{ ft.}$$

Alternatively by Art. 20,

$$\frac{30}{2{,}240}=\frac{f}{g}$$

$$f=\frac{30\times 32{\cdot}2 \text{ ft./sec.}^2}{2240}=0{\cdot}4312 \text{ ft./sec./sec.}$$

Time in seconds to reduce speed by 44 ft./sec.

$$=\frac{44 \text{ ft./sec.}}{0\cdot4312 \text{ ft./sec.}^2}$$

$$=102\cdot1 \text{ sec.}$$

Distance travelled in this time
at average speed of 22 ft./sec. $=22$ ft./sec. $\times 102\cdot1$ sec.
$=2{,}244$ ft. (as before)

Example 2. Work Example 1, Art. 23, from the gain in kinetic energy.

Initial speed, 20 m.p.h. $=\frac{88}{3}$ ft./sec. $=29\cdot3$ ft./sec.

Final speed, 50 m.p.h. $=\frac{5}{6}\times 88$ ft./sec. $=\frac{220}{3}$ ft./-sec.
$=73\cdot3$ ft./sec.

Final kinetic energy $=\frac{1{,}800}{2\times32\cdot2}\times73\cdot2^2$ ft.-lb.

Initial kinetic energy $=\frac{1{,}800}{2\times32\cdot2}\times29\cdot3^2$ ft.-lb.

Gain of kinetic energy $=\frac{1{,}800}{2\times32\cdot2}(73\cdot3^2-29\cdot3^2)$.
$=126{,}300$ ft.-lb.

Distance travelled in 10 sec. at average speed of $\frac{1}{2}(73\cdot3+29\cdot3)$ or $51\cdot3$ ft./sec. is $51\cdot3$ ft./sec. $\times 10$ sec. $=$ 513 ft.

$$\text{Force exerted}=\frac{\text{work done}}{\text{distance}}=\frac{\text{gain in kinetic energy}}{\text{distance}}$$

$$=\frac{126{,}300 \text{ ft.-lb.}}{513 \text{ ft.}}$$

$$=246 \text{ lb. (as before).}$$

Example 3. A body weighing 50 kilogrammes is moving with a velocity of 5 metres per sec. What is its kinetic energy and what constant force would be required to bring it to rest in 10 metres ?

$$\text{Kinetic energy} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\frac{50{,}000 \text{ gram}}{1} \times 500^2\left(\frac{\text{cm.}}{\text{sec.}}\right)^2$$

$$= 625 \times 10^7 \text{ ergs (or cm.-dynes)}$$

$$= 625 \text{ joules.}$$

Force to do 625 joules of work in 1,000 cm.

$$= \frac{625 \times 10^7 \text{ cm.-dynes.}}{1{,}000 \text{ cm.}}$$

$$= 625 \times 10^4 \text{ dynes}$$

$$= \frac{625 \times 10^4}{981} \text{ grammes or } 6{\cdot}37 \text{ kg.}$$

$$= \frac{6{\cdot}37 \text{ kg.}}{0{\cdot}453 \text{ kg./lb.}} = 14 \text{ lb.}$$

Checking this, the average speed in coming to rest is 2·5 metres per sec. and the time is 10 metres/2·5 metres per sec., which is 4 sec.

$$\text{Deceleration} = \frac{500 \text{ cm. per sec.}}{4 \text{ sec.}} = 125 \text{ cm./sec./sec.}$$

Using the method of Art. 20 and gravitational units,

Force required in grammes for this deceleration

$$= 50{,}000 \text{ gm. force} \times \frac{125 \text{ cm./sec.}^2}{981 \text{ cm./sec.}^2}$$

$$= 6{,}370 \text{ g. or } 6.37 \text{ kg. (as before)}$$

Example 4. If the motor vehicle in Example 1 is brought to rest by brakes, find how much heat will be produced.

The whole of the kinetic energy will be converted into heat at the rate of 778 ft.-lb. per B.Th.U. Hence,

$$\text{Heat produced} = \frac{67{,}340 \text{ ft.-lb.}}{778 \text{ ft.-lb./B.Th.U.}} = 86{\cdot}6 \text{ B Th.U.}$$

Example 5. A body weighing 10 tons has exerted upon it a force P which varies in magnitude as shown below, with the distance x ft., which the body has moved from rest, the direction being constant.

x ft.	0	5	10	15	20	25
P lb.	3500	3200	2900	3100	3000	2500

The motion of the body is opposed by a constant frictional resisting force of 800 lb. Find the speed of the body after it has moved 25 ft. from rest.

The final kinetic energy of the body is equal to the work done in 25 ft. by the effective force exerted in accelerating the body. For a force varying in this way, we can use a diagram of work to find the energy given to the body. We could subtract 800 lb. from each value of P before plotting the values, but in Fig. 23 the diagram is shown with full values of P along the graph EFG and a reduction of the energy to a base line AB at a level of 800 lb. The area below AB, viz. ABCD, represents the energy spent in overcoming friction and converted into heat, and that above AB, viz. AEFGB, represents the kinetic energy given to the body from

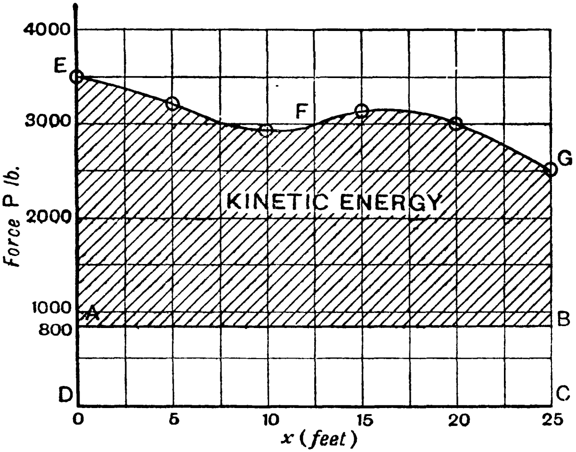

FIG 23.- Kinetic energy problem.

whatever source of energy exerts a force P through the 25-ft. travel. The scales of Fig. 23 were originally 1,000 lb. to 1 in. and 5 ft. to 1 in., but the diagram is here shown half size. On the full scale, 1 sq. in. area represents 1,000 lb. × 5 ft. = 5,000 ft.-lb. On square paper, ruled, say, to 1/10 in., the area can readily be found by counting the small squares. The area AEFGB is about 11·2 sq. in. when drawn to full scale and hence the work done in acceleration (and consequently the kinetic energy) is

$$11{\cdot}2 \text{ sq. in.} \times 5{,}000 \text{ ft.-lb./sq. in.} = 56{,}000 \text{ ft.-lb.}$$

Then if v is the speed of the body in feet per second after moving 25 ft.,

$$\text{K.E.} = \frac{10 \times 2{,}240 \text{ lb.} \times v^2}{2 \times 32{\cdot}2 \text{ ft./sec.}^2} = 56{,}000 \text{ ft.-lb.}$$

$$v^2 = \frac{2 \times 32{\cdot}2 \times 56{,}000}{22{,}400} \text{ ft.}^2/\text{sec.}^2$$

$$= 161 \text{ (ft./sec.)}^2$$

$$v = \sqrt{161 \text{ (ft./sec.)}^2} = 12{\cdot}7 \text{ ft./sec.}$$

We might similarly find the speed from the kinetic energy at other distances, such as 5, 10, 15, and 20 ft., and plot a graph of v on a base representing values of x.

32. Exchanges of Energy. In any action involving a transfer of energy, a ruling principle which assists computation is that of the Conservation of Energy. In mechanical operations, frequently there is friction and some energy is converted into heat. But in others, friction may be negligibly small and we may have potential energy converted into kinetic energy and this again converted into potential energy. And so long as friction is negligible we can in calculation employ the principle that the total mechanical energy, say, kinetic plus potential energy, remains constant. Thus, if we can estimate, say, the loss in one, we know the gain in the other. For example, if a body runs down an inclined plane with negligible friction its gain in kinetic energy is equal to its loss of potential energy, which is equal to its weight multiplied by its change in level, i.e. its vertical fall. And from its gain in kinetic energy we can calculate its gain in speed. Or if a body suspended by a flexible string, or on a hinge, swings like a pendulum, it repeatedly exchanges potential into kinetic energy as it swings downwards and exchanges kinetic into potential energy as it swings upward until at the top of its stroke its energy is wholly potential and its kinetic energy is nil. Examples will make the principle clear.

Example 1. *Frictionless incline.* If a body of weight W lb. slides down an incline of length l ft., height h ft., and base b ft. (Fig. 24) from rest at the top, find its speed at the base.

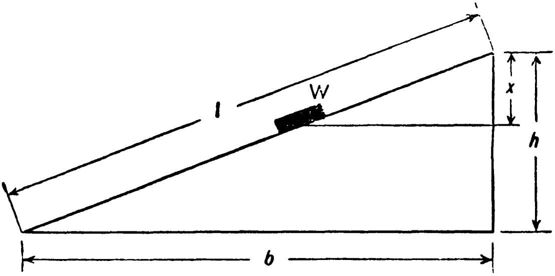

FIG. 24.—Increase of kinetic energy down slope.

When it reaches the base it has lost potential energy W × h ft.-lb. Hence its kinetic energy is Wh ft.-lb. or

$$\frac{1}{2}\frac{\text{W lb. } v^2}{g \text{ ft./sec.}^2} = \text{W}h \text{ ft.-lb.}$$

where v is the speed of the body in feet per second. Hence

$$v^2 = 2gh \text{ (ft./sec.)}^2$$

$$\text{or } v = \sqrt{2gh} \text{ ft./sec.}$$

The result, $v^2 = 2gh$, for a vertical fall, is merely a particular case of the more general result (3) of Art. 8. But it is here very easily reached by the principle of the Conservation of Energy. Moreover, the principle makes it clear that, provided there is no friction, the speed v is the same (viz. $\sqrt{2gh}$) after a descent through a height h, whatever the steepness of the incline ; it is the same as for a free vertical fall. ((1) Art. 10.)

When the body has descended through a vertical distance x ft. its kinetic energy is Wx ft.-lb. and its speed $\sqrt{2gx}$ ft. per sec. If it is able to descend h ft. vertically in all, it may be said to have still a potential energy $W(h-x)$ ft.-lb. But this assumes the level of the base of the incline as a datum level at which the body is regarded as having no potential energy.

Example 2. A metal ball weighing 10 lb. can swing in a vertical circle of 5 ft. radius at the end of a cord or light rod. It is released from a position A (Fig. 25) and swings about a centre O. Find its kinetic energy and its speed (1) when it has reached a position B, the string having turned through an angle of 30°, and (2) when it is at C, vertically below the centre of motion O.

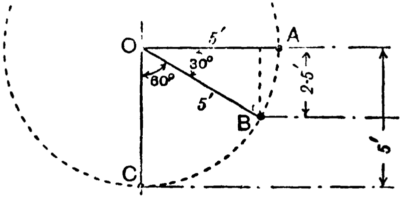

FIG. 25. Swinging ball problem.

Here again there is no loss of energy and the kinetic energy is equal to the loss of potential energy.

(1) At B the ball is 5/2 ft. below A. Hence

$$\text{its K.E.} = \text{loss of potential energy}$$
$$= 10 \text{ lb.} \times 2{\cdot}5 \text{ lb.} = 25 \text{ ft.-lb.}$$

or $$\frac{1}{2}\,\frac{10 \text{ lb.}}{32{\cdot}2 \text{ ft./sec.}^2}\,v^2 = 25 \text{ ft.-lb.}$$

$$v^2 = \frac{25 \times 2 \times 32{\cdot}2}{10} = 161 \text{ (ft./sec.)}^2$$

$$v = \sqrt{161} \text{ ft./sec.} = 12{\cdot}7 \text{ ft./sec.}$$

(2) At C, the ball is 5 ft. below A. Hence its K.E. is 10 lb. × 5 ft. = 50 ft.-lb.

$$\frac{1}{2}\frac{10}{3{\cdot}22}v^2 = 50 \text{ ft.-lb.}$$

$$= \sqrt{322} \text{ ft./sec.} = 17{\cdot}9 \text{ ft./sec.}$$

Example 3. A thick metal disc weighing 10 lb. is attached to a spring of negligible weight and can move horizontally to and fro (*see* Fig. 26). The spring yields at the rate of 1 in. per 20 lb. pull or thrust exerted upon it. If the disc is displaced 3 in. axially from its position of equilibrium, what will be its speed in passing through this position again after release ?

What will be its kinetic energy when midway between the extreme (displaced) position and the equilibrium position ? Draw a diagram to show the kinetic and the elastic strain-energy in all positions.

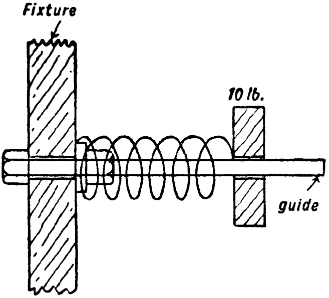

FIG. 26.—Oscillating disc.

Since the force required to strain the spring elastically is at the rate of 20 lb. per in., to displace the disc 3 in. will require a force rising from zero proportionally to the displacement, up to a maximum of 3 in. × 20 lb. per in. = 60 lb. The diagram of work is shown in Fig. 27, *ab* being the distance the disc is moved, viz. 3 in., and vertical co-ordinates, to the line *ac*, being the forces, reaching a maximum of *cb*, which represents 60 lb. to scale. When the disc is released the spring drives it back to its original position at *a*, where there is no strain and no force exerted, and in so doing the spring gives up its strain energy by imparting kinetic energy to the disc. At *a*, the disc has acquired its maximum speed and kinetic energy, and it overshoots this point and gives up its kinetic energy in straining the spring again. If the first displacement was a stretch of the spring, the next will be a shortening or compression, and vice versa. The disc will oscillate about a mid-position *a* in which it will be moving fastest and have maximum kinetic energy. At the ends of its travel it will have zero speed and all the energy will have returned to the elastically strained spring. Only half the motion, that to the right of *a*, is shown in Fig. 27.

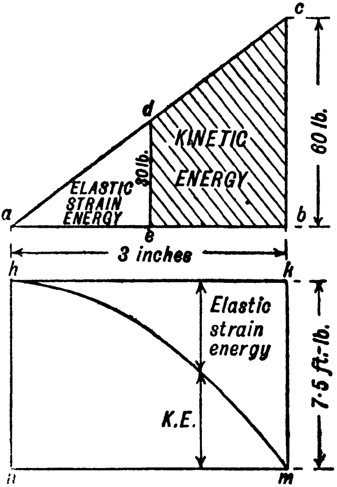

FIG. 27.—Energy changes in oscillation.

The elastic strain energy at *b* will be represented by the triangular work diagram *abc*, which represents by its area (½ base × vertical height),

$$\tfrac{1}{2}\times 3\text{ in.}\times 60\text{ lb.} = 90\text{ in.-lb.}$$
$$= 7{\cdot}5\text{ ft.-lb.}$$

When passing through the unstrained position *a* again the whole of its energy will be kinetic energy. Hence if *v* is its speed in feet per second,

$$\frac{1}{2}\frac{W}{g}v^2 \text{ or } \frac{1}{2}\,\frac{10\text{ lb.-force}}{32{\cdot}2\text{ ft./sec.}^2}\times v^2 = 7{\cdot}5\text{ ft.-lb.}$$

$$v^2 = \frac{7{\cdot}5\times 32{\cdot}2\times 2}{10}\text{ (ft./sec.)}^2 = 48{\cdot}3\text{ (ft./sec.)}^2$$

$$v = \sqrt{48{\cdot}3}\text{ ft./sec.} = 6{\cdot}95\text{ ft./sec.}$$

When passing through *e*, midway between *a* and *b*, the straining force will have fallen to 1·5 ×20 lb. =30 lb.

and the strain energy of the spring will be represented by the triangular work diagram *aed*, representing

$$\tfrac{1}{2}\times 1{\cdot}5 \text{ in.} \times 30 \text{ lb.} = 22{\cdot}5 \text{ in.-lb.}$$
$$= 1{\cdot}875 \text{ ft.-lb.}$$

and the remainder, $7{\cdot}5 - 1{\cdot}875 = 5{\cdot}625$ ft.-lb., will be kinetic energy, represented by the area *ebcd*. (We may check that this area is $eb \times \tfrac{1}{2}(ed + bc)$ and represents

$$1{\cdot}5 \text{ in.} \times \frac{30+60}{2} \text{ lb.} = 1{\cdot}5 \times 45 \text{ in.-lb.} = 67{\cdot}5 \text{ in.-lb.}$$
$$= 5{\cdot}625 \text{ ft.-lb.})$$

And similarly when the disc is at any point between *a* and *b* the area under *ac*, and to the right of the point, represents the kinetic energy, and the area to the left represents the elastic strain energy.

In the lower part of Fig. 27 ordinates representing the triangular areas (or the strain energy) have been set off downwards from the line *hk*, reaching a maximum *km*, representing 7·5 ft.-lb. at *b*. The distances downward from this graph *hm* to *nm* (representing 7·5 ft.-lb. minus the strain energy) show the kinetic energy for each position.

The *total* energy remains at 7·5 ft.-lb. so long as the spring is perfectly elastic and the motion is unrestricted by any frictional forces, a condition which is, of course, only imperfectly realised in practice, though a mass attached to a spring will oscillate with small loss of energy for a long time under favourable conditions. To simplify the problem the disc has been taken as moving horizontally. But a nearly frictionless arrangement could more easily be arranged by hanging a load on a helical spring with its axis vertical. Here we should have changes in kinetic energy, gravitational potential energy and elastic strain energy. But the *effective* force on the disc being the difference between the downward pull of the earth and the upward pull of the spring would be as in the case illustrated, and the change in kinetic energy in oscillation would be the same. This type of motion is a very important one for study at a later stage.

✓Example 4. A pair of pulley blocks have a velocity ratio of 4 to 1 and both effort and load take the form of weights falling and rising respectively. Experiment shows that a load of 42 lb. is just lifted with no acceleration by a (falling weight) effort of 15 lb. instead of the 10·5 lb. which would be required if there were no friction. (In other words, the mechanical efficiency 42/(4 ×15) is 70 per cent.) If the falling weight (effort) is now increased to 20 lb, and the energy absorbed by friction is assumed to remain constant, find the speed of the effort-weight after it has fallen 10 ft. (neglect the inertia of the pulley blocks). Find also the tension in the cord or rope to which the 20-lb. weight is attached.

Work done by effort of 20-lb. weight

$$=20 \text{ lb.} \times 10 \text{ ft.} = 200 \text{ ft.-lb.}$$

Increase of potential energy of load

$$=42 \text{ lb.} \times 2{\cdot}5 \text{ ft.} = 105 \text{ ft.-lb.}$$

Frictional work lost as heat as in unaccelerated lift

$$\frac{30}{100} \times 15 \text{ lb.} \times 10 \text{ ft.} = 45 \text{ ft.-lb.}$$

Hence kinetic energy of the two moving weights is given by

$$\text{kinetic energy} = 200 - 105 - 45 = 50 \text{ ft.-lb.}$$

(which might also be deduced by taking the energy of the additional 5 lb. added, falling through 10 ft., viz. 5 lb. ×10 ft.).

Let v = speed of 20-lb. falling weight in feet per second, then,

$\frac{1}{4}v$ = speed of 42-lb. rising load.

$$\text{Total K.E.} = \tfrac{1}{2}\frac{20}{32{\cdot}2}v^2 + \tfrac{1}{2}\frac{42}{32{\cdot}2}(\tfrac{1}{4}v)^2$$

$$= \frac{1}{64{\cdot}4 \text{ ft./sec.}^2}\left(20 + \frac{42}{16}\right) \text{lb.-force} \times v^2$$

$$= \frac{22{\cdot}625 \text{ lb.-force}}{64{\cdot}4 \text{ ft./sec.}^2} \times v^2$$

and this is equal to the 50 ft.-lb., as shown above.

Hence

$$v^2 = \frac{50 \text{ ft.-lb.} \times 64{\cdot}4 \text{ ft./sec.}^2}{22{\cdot}625 \text{ lb.-force}} = 142{\cdot}3 \text{ (ft./sec.)}^2$$

$$v = 11{\cdot}93 \text{ ft./sec.}$$

Let T be the tension in lb. in the cord applying the effort. Then the net downward force on the 20-lb. weight is (20 — T) lb., and this force acting through 10 ft. gives the 20-lb. weight the known speed 11·93 ft./sec. (or $v^2 = 142{\cdot}3$ (ft./sec.)^2); hence equating the work done by it to the gain of K.E. by the 20-lb. weight

$$(20 - \text{T}) \text{ lb.} \times 10 \text{ ft.} = \tfrac{1}{2}\frac{20 \text{ lb.-force} \times 142{\cdot}3 \text{ (ft./sec.)}^2}{32{\cdot}2 \text{ ft./sec.}^2}$$

$$20 - \text{T} = \frac{142{\cdot}3}{32{\cdot}2} \text{ lb.} = 4{\cdot}42 \text{ lb.}$$

$$\text{T} = 20 - 4{\cdot}42 = 15{\cdot}58 \text{ lb.}$$

The number of significant figures given may seem excessive, but here they give an interesting check upon the work. For we may note that of the 5 lb. of added weight which gives 50 ft-lb. of K.E., 4·42 lb. gives 4·42 lb. ×10 ft. =44·2 ft.-lb. of K.E. to the 20-lb. weight, and 0·58 lb. (the excess over the original 15 lb.) gives 0·58 lb. ×10 ft. =5·8 ft.-lb. of K.E. to the 42-lb. weight. The ratio of these two is 44·2/5·8 =7·6, which is also that of 20 lb. to 42/16 lb., namely the ratios of the K.E.'s acquired when the 42-lb. weight is moving at ¼ of the speed of the 20-lb. weight.

✓**33. Kinetic Energy of Rotation.** We have seen that a force exerted upon a body in the direction of its motion does work upon it and gives it kinetic energy. Similarly, a torque exerted upon a body in a plane perpendicular to an axis about which it is rotated does work and gives it kinetic energy, not of flight, but of spin or rotation.

Suppose, as in Art. 27, the body weighing W lb. is made up of small parts weighing W_1 lb., W_2 lb., W_3 lb., etc., at distances r_1 ft., r_2 ft., r_3 ft., etc., from the axis of rotation. If the angular velocity of the body about the axis is ω radians per second, then ωr_1, ωr_2, ωr_3, are the speeds in feet per second of the various parts. Then the total kinetic energy of the parts is

$$\text{K.E.}=\tfrac{1}{2}\frac{W_1}{g}(\omega r_1)^2+\tfrac{1}{2}\frac{W_2}{g}(\omega r_2)^2+\tfrac{1}{2}\frac{W_3}{g}(\omega r_3)^2+\text{etc., ft.-lb.}$$

$$=\tfrac{1}{2}\omega^2\left\{\frac{W_1}{g}r_1^2+\frac{W_2}{g}r_2^2+\frac{W_3}{g}r_3^2+\text{etc.}\right\}\text{ft.-lb.}$$

$$=\tfrac{1}{2}\frac{W}{g}k^2\omega^2\text{ ft.-lb.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where, as in (5), Art. 27, k^2 is the "weighted" mean of the squares of the distances of the various parts in feet from the axis of rotation (or simply the mean, if all the parts are of equal weight) and k is called the radius of gyration of the body about that axis. $\frac{W}{g}k^2$ is then the measure of the rotary inertia about this axis. The radius k is therefore that at which the mass of the wheel may be regarded as being concentrated.

We could arrive at this result directly from (5) of Art. 27. For if in a time t sec. the constant torque of T lb.-ft. turns the body through an angle θ radians, increasing the angular velocity from ω_1 to ω_2 radians per sec., $\theta=t\times$average angular velocity$=\frac{1}{2}t(\omega_2+\omega_1)$ and $\alpha=\frac{\omega_2-\omega_1}{t}$ radians per sec. per sec. Then from (5), Art. 27,

$$T=\frac{W}{g}k^2\alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

so that

$$T\theta=\frac{W}{g}k^2\alpha\theta\text{ ft.-lb.}$$

$$=\frac{W}{g}k^2\times\frac{\omega_2-\omega_1}{t}\times\frac{t}{2}(\omega_2+\omega_1)$$

$$=\tfrac{1}{2}\frac{W}{g}k^2(\omega_2^2-\omega_1^2) \quad . \quad . \quad . \quad . \quad (3)$$

That is, the work done by the torque, namely $T\theta$ ft.-lb., is the increase in kinetic energy. And if $\omega_1=0$, the body being initially at rest, the kinetic energy at ω_2 is,

$$\tfrac{1}{2}\frac{W}{g}k^2\omega_2^2\text{ ft.-lb.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The expression (3) shows that the gain of kinetic energy is proportional to the difference in the square of the final and initial speeds. Thus, if a flywheel is running at N r.p.m. and its speed is increased by 1 per cent. to 1·01N, its kinetic energy will be increased in the ratio $(1{\cdot}01N)^2/N^2 = 1{\cdot}02$ very closely, that is, its kinetic energy will be increased by 2 per cent. And for any small change in kinetic energy, the percentage change in speed will only be *half* that of the change in kinetic energy. The essential function of a flywheel is to take up excess energy in a variable rate of supply and to give it out when the rate of supply is below the average (see Arts. 68 and 75).

Example 1. A flywheel weighs 1,000 lb. and has a radius of gyration of 2·5 ft. An average driving torque of 150 lb.-ft. in excess of any frictional torque is exerted upon it. Find the speed of the wheel after it has made 5 revolutions from rest. How long will it take to attain this speed?

The energy given to it by an average torque of 150 lb.-ft. in 5 revolutions will be the torque × angle turned through in radians.

$$\text{Angle in radians} = 5 \text{ revs.} \times 2\pi \text{ radian/rev.}$$

$$= 10\pi \text{ radians.}$$

Hence,

$$\text{kinetic energy of wheel} = 150 \text{ lb.-ft.} \times 10\pi \text{ radians.}$$

$$= 1{,}500\pi \text{ ft.-lb.}$$

$$= 4{,}712 \text{ ft.-lb.}$$

If ω is its angular velocity in radians/sec.,

$$\tfrac{1}{2} \cdot \frac{1{,}000 \text{ lb.-force}}{32{\cdot}2 \text{ ft./sec.}^2} \times \frac{2{\cdot}5^2}{1} \text{ft.}^2 \times \omega^2 = 4{,}712 \text{ ft.-lb.}$$

$$\omega^2 = \frac{4{,}712 \times 32{\cdot}2 \times 2}{1{,}000 \times 6{\cdot}25} \left(\frac{\text{radians}}{\text{sec.}}\right)^2$$

$$= 48{\cdot}6 \text{ (radians/sec.)}^2$$

$$\omega = \sqrt{48{\cdot}6} = 6{\cdot}97 \text{ radians/sec.}$$

$$\text{Or the speed} = \frac{6{\cdot}97 \text{ radians/sec.} \times 60 \text{ sec./min.}}{2\pi \text{ radians/rev.}}$$

$$= 68{\cdot}1 \text{ r.p.m.}$$

$$\text{Time taken} = \frac{\text{angle turned through}}{\text{average angular velocity}} = \frac{5 \times 2\pi \text{ radians}}{\frac{6{\cdot}97}{2} \text{ radians/sec.}}$$

$$= 9{\cdot}0 \text{ sec.}$$

Example 2. A flywheel weighing 2,000 lb. is attached to a machine which overcomes a periodic resistance doing work, and receiving energy from the flywheel. The speed of the wheel drops from 150 r.p.m. to 148 r.p.m. in giving out 1,800 ft.-lb. of energy. Find the kinetic energy of the wheel at 150 r.p.m. and its radius of gyration.

If ω_1 = angular velocity in radians per second at 150 r.p.m. and ω_2 = angular velocity in radians per second at 148 r.p.m.

$$\text{K.E. at 150 r.p.m.} = \tfrac{1}{2}\frac{W}{g}k^2\omega_1^2$$

$$\text{K.E. at 148 r.p.m.} = \tfrac{1}{2}\frac{W}{g}k^2\omega_2^2.$$

$$\text{Loss of K.E.} = \tfrac{1}{2}\frac{W}{g}k^2(\omega_1^2 - \omega_2^2) = 1{,}800 \text{ ft.-lb.}$$

$$\text{K.E. at 150 r.p.m.} = 1{,}800 \text{ ft.-lb.} \times \frac{\omega_1^2}{\omega_1^2 - \omega_2^2}$$

and since $\dfrac{\omega_1^2}{\omega_1^2 - \omega_2^2} = \dfrac{N_1^2}{N_1^2 - N_2^2}$ where N stands for r.p.m.,

$$\text{K.E. at 150 r.p.m.} = 1{,}800 \text{ ft.-lb.} \times \frac{150^2}{150^2 - 148^2}.$$

$$= 1{,}800 \times \frac{22{,}500}{596} \text{ ft.-lb.}$$

$$= 67{,}970 \text{ ft.-lb.}$$

From this we can find the radius of gyration, for we know the mass and the speed of the wheel.

$$\text{At 150 r.p.m., } \omega_1 = \frac{150 \text{ r.p.m.} \times 2\pi \text{ radians/rev.}}{60 \text{ sec./min.}}$$

$$= \frac{150 \times 2\pi}{60} \text{ radians/sec.}$$

$$= 5\pi \text{ radians/sec.}$$

$$\text{K.E.} = \tfrac{1}{2}\frac{W}{g}k^2\omega_1^2 = \tfrac{1}{2}\frac{20{,}00 \text{ lb.-force.}}{32{\cdot}2 \text{ ft./sec.}^2} \times \frac{25\pi^2(\text{radians/sec})^2 k^2}{1}$$

$$= \frac{2{,}000 \times 25\pi^2}{64{\cdot}4} \frac{\text{lb.-force.}}{\text{ft.}} \times k^2$$

$$= 7{,}663 \,.\, k^2 \frac{\text{lb.-force}}{\text{ft.}}.$$

But this K.E. has been found to be 67,970 ft.-lb. Hence,

$$7{,}663k^2 \frac{\text{lb.-force}}{\text{ft.}} = 67{,}970 \text{ ft.-lb.}$$

$$k^2 = \frac{67{,}970 \text{ ft.-lb.}}{7{,}663 \text{ lb./ft.}} = 8{\cdot}87 \text{ ft.}^2$$

$$k = \sqrt{8{\cdot}87} \text{ ft.} = 2{\cdot}98 \text{ ft.}$$

Example 3. A flywheel weighing 5 tons has a radius gyration of 3·5 ft. and a speed of 200 r.p.m. Find its kinetic energy. To what speed will it fall if 100,000 ft.-lb. of energy are taken from it?

$$\text{Its angular velocity} = \frac{200 \text{ r.p.m.} \times 2\pi \text{ radians/rev.}}{60 \text{ sec./min.}}$$

$$= \frac{400\pi}{60} \text{ radians/sec.}$$

$$= 20{\cdot}94 \text{ radians/sec.}$$

$$\text{K.E.} = \tfrac{1}{2}\frac{5 \times 2{,}240 \text{ lb.-force}}{32{\cdot}2 \text{ ft./sec.}^2} \times (20{\cdot}94)^2 \text{ (radians/sec.)}^2 \times (3{\cdot}5)^2 \text{ ft.}^2$$

$$= 930{,}500 \text{ ft.-lb.}$$

K.E. after losing 100,000 ft.-lb. = 830,500 ft.-lb.

Let N then be the speed in revolutions per minute. Since the K.E. is proportional to the square of the speed,

$$\frac{N^2}{200^2}=\frac{830,500}{930,500}$$

$$N^2=200^2\times\frac{8,305}{9,305}=200^2\times0{\cdot}8928$$

$$N=200\times\sqrt{0{\cdot}8928}\text{ r.p.m.}=200\times0{\cdot}9449\text{ r.p.m.}$$

$$=189\text{ r.p.m.}$$

34. Impact and Impulsive Forces. When force is exerted between two bodies there is always deformation of both even if it be too small to be appreciable. If this is within the limits of perfect elasticity, there is no loss of mechanical energy by conversion of any portion of it into heat. But in most cases elasticity is imperfect and in some it is almost entirely absent.

When two bodies collide, the duration of the mutual contact during which they exert force upon each other may be very brief, but the forces may be large and the impulse (i.e. the product of force and the time) may be considerable. Such forces, exerted during impact are called impulsive forces and, being large, they often involve inelastic strains of the bodies and the conversion of mechanical energy into heat. The deformation of one body may of course be much greater than that of the other, as, for instance, when a tennis ball strikes a cement floor. In this case by dropping a ball vertically it will be apparent that there is some loss of energy, for the ball will not rebound to the height from which it has been dropped.

Impacts of special interest are (1) those in which there is no appreciable elasticity and no rebound of one colliding body from the other, and (2) those in which there is perfect elasticity and a rebound with no loss of energy.

35. Inelastic Impacts. When there is no appreciable elastic action between two bodies in collision they remain in contact with a common velocity. Such a case was that of a bullet fired into a block of wood (see Example 1, Art. 26).

A bullet weighing 1 oz., travelling at 2,406 ft./sec., hits a stationary block of wood which then moves at 6 ft./sec. in conformity with the principle of the conservation of momentum. But most of the kinetic energy of the bullet is lost, being converted into heat. For

$$\text{K.E. before impact} = \tfrac{1}{2}\frac{\text{lb. force}}{16} \times \frac{2{,}406^2\ (\text{ft./sec.})^2}{32{\cdot}2\ \text{ft./sec.}^2}$$

$$= 5{,}622\ \text{ft.-lb.}$$

$$\text{K.E. after impact} = \tfrac{1}{2}\frac{25{\cdot}06\ \text{lb.-force}}{32{\cdot}2\ \text{ft./sec.}^2} \times \frac{6^2(\text{ft./sec.})^2}{1}$$

$$= 14{\cdot}0\ \text{ft.-lb.}$$

Thus all the 5,622 ft.-lb. of energy except 14·0 ft.-lb. is converted into heat by the impact.

In other cases where the disparity in mass of the two bodies concerned in an inelastic impact is not so great, the proportion of kinetic energy lost will not be so great but it will generally be considerable. It is important to remember that in such cases the *kinetic energy is not constant or conserved.* The momentum is conservative and so is the total energy ; but when some of it becomes heat, there is a loss of kinetic energy. Mistakes are often made by failure to appreciate this. Constant mechanical energy is a principle which can be used when a body is oscillating under the alternating thrust and pull of a perfectly elastic spring, as in Example 3 of Art. 32, but not in cases of inelastic impact.

Example. **A cylindrical casting, with its axis vertical, weighing 1 lb., falls vertically through 5 ft. and impinges upon a similar cylindrical casting weighing 3 lb., supported on a light spring, as diagrammatically indicated in Fig. 28. With what speed do the two bodies move downward before the retarding effect of the spring is appreciable and how much energy is converted into heat in the impact ?**

Potential energy converted to kinetic energy in the fall of 1 lb. through 5 ft. is 1 lb. ×5 ft. =5 ft.-lb.

$$\text{K.E.}=\tfrac{1}{2}\frac{1\text{ lb.-force}}{32{\cdot}2\text{ ft./sec.}^2}\times v_1{}^2=5\text{ ft.-lb.}$$

where v_1 is the speed of the 1-lb. casting at impact.

Hence

$$\begin{aligned} v_1{}^2 &= 5\times 2\times 32{\cdot}2\ (\text{ft./sec.})^2 \\ &= 322\ (\text{ft./sec.})^2 \\ v_1 &= \sqrt{322}\text{ ft./sec.} = 17{\cdot}94\text{ ft./sec.} \end{aligned}$$

and since the momentum after impact = momentum before impact, using pound mass and feet per second units

$$1\text{ lb.}\times 17{\cdot}94\text{ ft./sec.}=(1+3)\text{ lb.}\times v_2$$

where v_2 is the speed of the combined bodies just after impact, hence

$$v_2=\frac{17{\cdot}94}{4}=4{\cdot}485\text{ ft./sec.}$$

Kinetic energy just after impact is given by

$$\begin{aligned}\text{K.E.} &= \tfrac{1}{2}\frac{4\text{ lb.force}}{32{\cdot}2\text{ ft./sec.}^2}\times 4{\cdot}485^2\ (\text{ft./sec.})^2 \\ &= 1{\cdot}25\text{ ft.-lb.}\end{aligned}$$

Loss of K.E. at impact = 5 ft.-lb. − 1·25 ft.-lb. = 3·75 ft.-lb., which is $\frac{3}{4}$ of the initial amount, and this is converted into heat.

It may be noticed that in the problem (Example 1, Art. 26 and Art. 35 above) of the 1-oz. bullet fired into a block weighing 400 oz. the surviving K.E. was 1/401 part of the initial or 400/401 parts were converted into heat. And in the example of masses of 1 lb. and 3 lb., $\frac{1}{4}$ of the initial K.E. survived and $\frac{3}{4}$ was converted into heat. This is a general result which may be left as an exercise to prove that if a mass m_1 impinges on a stationary inelastic mass m_2, which has no other force acting upon it, the K.E. lost as heat is $\frac{m_2}{m_1+m_2}$ of the original K.E. and the fraction surviving after impact is $\frac{m_1}{m_1+m_2}$.

Average Force. During any action, such as collision, between two bodies, there will generally be a varying magnitude to the equal forces exerted by each on the other. In most cases it is exceedingly difficult to measure such a force and to know how it varies. Sometimes in purely academic problems it is assumed to be constant; it can rarely be so in fact. From the circumstances we may be able to state an average force. But we must understand what we mean by an average force; it may be a time average or a space (or distance) average. If we know the change of momentum of a body due to a collision, we know that the impulse is equal to this (Art. 24). And if we know or estimate the time during which the force causing the change of momentum is exerted, we can calculate the *time-average* force as the impulse or change of momentum divided by that time. Thus, if a hammer moving at a known speed strikes a chisel and comes to rest and contact lasts, say, 0·01 second, the momentum given to the chisel divided by the 0·01 sec. would give the time-average force of the blow or of the mutual force exerted between hammer and chisel.

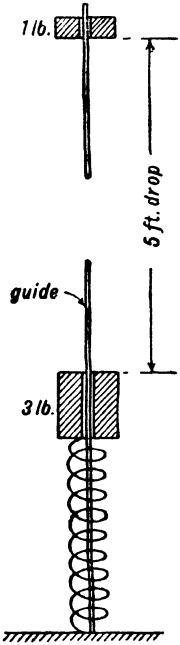

FIG. 28.—An inelastic impact.

And to illustrate a space average we might consider the punching of metal or driving of a nail or a pile If we know the distance through which the force is exerted in penetration and the energy expended, whether kinetic or otherwise, we can say that the energy expended or work done in the action, divided by the distance through which the resistance to penetration is exerted, is the ***space average*** **of the mutual forces exerted.**

36. Elastic Impacts. One case of elastic impact is of special interest at this stage for application later in the book (Arts. 55 and 57). The molecules of a gas are conceived as perfectly elastic bodies of more or less spherical shape and we shall require to know the effect of their impact on the stationary wall of a vessel containing them and also upon a moving piston receding from an advancing molecule or advancing towards it.

We can simplify the problem by considering only impacts of a more or less spherical body on a flat surface and only the motion normal (i.e. perpendicular) to the flat surface. Motion in any other direction can be resolved into two components, one normal to the flat surface and the other at right-angles to the normal, i.e. parallel to the flat surface. This latter component is unaffected by the impact, for there is no force in its direction and therefore, according to the laws of motion, speed and momentum in that direction remain unchanged. Hence only motion normal to the surface need be considered. As we are here considering a single molecule of exceedingly small mass (see footnote to Art. 50), we cannot well insert numerical values, but a brief investigation in algebraic symbols will later clarify what occurs in a gas, and to abbreviate the work we will employ the symbol m for mass: this will serve for any system of units.

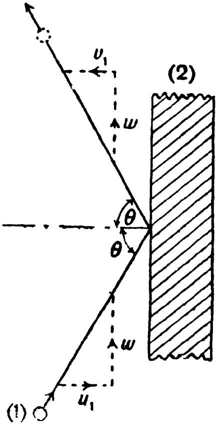

FIG. 29.—Elastic impact.

Suppose that a very small elastic body, such as a sphere denoted by (1) in Fig. 29, impinges on a much larger and much more massive body (2). Let m_1 be the mass of the small body (1) and m_2 that of the more massive body (2). Let u_1 and v_1 be the velocities of body (1) normal to the flat surface of body (2) before and after impact respectively and let u_2 and v_2 be the velocities in the same direction of the body (2) before and after the impact.

Then from the principle of the conservation of momentum (Art. 26),

total momentum before impact = total after impact

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad . \quad . \quad . \quad (1)$$

or rearranging the terms

$$m_1(v_1 - u_1) = -m_2(v_2 - u_2) \quad . \quad . \quad . \quad (2)$$

And since there is perfect elasticity in the impact, there is no loss of kinetic energy.

K.E. before impact = K.E. after impact

$$\tfrac{1}{2}\, m_1u_1^2 + \tfrac{1}{2}\, m_2u_2^2 = \tfrac{1}{2}\, m_1v_1^2 + \tfrac{1}{2}\, m_2v_2^2 \quad . \quad (3)$$

or rearranging the terms and multiplying by 2

$$m_1(v_1^2 - u_1^2) = -m_2(v_2^2 - u_2^2) \quad . \quad . \quad (4)$$

Then dividing this by (2)

$$v_1 + u_1 = v_2 + u_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

or

$$v_1 = -u_1 + v_2 + u_2 \quad . \quad . \quad (6)$$

Stationary Obstacle. If the mass m_2 is very large and the body (2) stationary so that $u_2 = 0$, then it is evident from (2) that v_2 is very small and if m_2 is many million times as great as m_1, v_2 will be negligibly small or zero. Then equation (5) or (6) becomes

$$v_1 = -u_1$$

That is, the normal velocity of rebound of body (1) from a heavy stationary body (2) is equal in magnitude to its normal velocity of approach. And if there is a component w at right-angles to the normal velocity this remains unaltered by the impact; and as shown in Fig. 29, the body (1) (like a ray of reflected light) has an inclination θ to the normal line after and before meeting the flat surface. But we are mainly concerned with the normal velocity only as if w were zero (as it may be).

Moving Obstacle. But the body (2) may not be stationary; it may be moving at a speed virtually unaffected by this impact of a minute body or, in other words, from equation (2) if m_2 is large $v_2 - u_2$ may be negligible so that $v_2 = u_2$. Let us call $v_2 = u_2 = v'$, say; this is the constant speed of the moving obstacle (such as an engine piston) both before and after the impact. Then from equation (5)

$$v_1+u_1=v_2+u_2=2v'$$

or $$v_1=-u_1+2v' \text{ or } -(u_1-2v') \quad . \quad . \quad . \quad (7)$$

In words this tells us that the final normal velocity in rebounding from the moving massive body is reversed in direction and diminished in magnitude by $2v'$ when the direction of v' is the same as that of u_1 (e.g. the case of a piston receding from an advancing molecule). But if the moving heavy body is advancing in the opposite direction to meet the lighter body (1), say $u_2=v_2=-v'$, then

$$v_1=-u_1-2v'=-(u_1+2v') \quad . \quad . \quad . \quad (8)$$

that is, the normal speed on rebound is *increased* in magnitude by $2v'$.

Transfer of Energy. The energy transferred from the body (1) to the body (2) is readily stated as the loss of K.E. by the body (1), viz. the initial K.E. minus the final K.E., or

$$\tfrac{1}{2}m_1u_1^2-\tfrac{1}{2}m_1v_1^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

But we might put it in another form, which brings out the push given to body (2) by body (1) during the impact and the work thereby done upon body (2); since $v_1=-(u_1-2v')$, the energy transferred as given in (9) is

$$\tfrac{1}{2}m_1u_1^2-\tfrac{1}{2}m_1(u_1-2v')^2=\tfrac{1}{2}m_1\{u_1^2-(u_1-2v')^2\}$$
$$=\tfrac{1}{2}m_1\times 4u_1v'=2m_1u_1v'$$

if we neglect the term which is multiplied by the square of v', and we can safely neglect this because the speed v' will be, say, only $\frac{1}{100}$ part of the speed of the molecule.

The energy transferred at the impact is, then,

$$2m_1u_1v'.$$

Suppose the impact occupies a time t sec. Then, since the total change of momentum from m_1u_1 to $-m_1u_1$ is $2m_1u_1$, $\dfrac{2m_1u_1}{t}$ is the average rate of change of momentum, which is the average force exerted in the direction of motion during the impact. And the distance moved by the moving piston is $t \times v'$. Hence

$$2m_1u_1v' = \frac{2m_1u_1}{t} \times v't$$

is the work done during the impact, which is of course equal to the transfer of energy.

In the case of an advancing piston the loss of K.E. by the molecule is

$$\tfrac{1}{2}m_1u_1^2 - \tfrac{1}{2}m_1v_1^2 = -2m_1u_1v'$$

which indicates that the loss of energy by the body (1) (or molecule) is negative, i.e. it *gains* energy of magnitude $2m_1u_1v'$ by the impact.

We may summarise the conclusions of this article thus: (1) If an elastic body strikes a stationary wall, it undergoes a change of momentum during contact and exerts a force but rebounds with unaltered speed, so that the magnitudes of the momentum and kinetic energy are both undiminished; (2) If it strikes a receding body, it rebounds with speed and momentum and kinetic energy diminished and does work upon the receding body. (3) If it strikes an advancing body, it rebounds with speed and momentum and kinetic energy all increased and work is done upon it by the advancing body. These conclusions will be useful in explaining the behaviour of a gas (Art. 55 (*b*) and Art. 57).

Summary of Chapter III

It follows from the relation of the force accelerating a body, the mass m and the acceleration, that from rest to a speed v, the work done is $\frac{1}{2}mv^2$ or $\frac{W}{2g}v^2$, which is the kinetic energy of the body.

The exchange or conversion of energy to or from other forms of energy enable us to calculate v^2, and therefore v, in such cases as a body sliding down an incline or swinging freely on a suspension or oscillating under the control of an elastic spring.

An accelerating torque gives a body rotational kinetic energy, the work done by the torque ($T\theta$) giving kinetic energy $\frac{1}{2}mk^2\omega^2$ or $\frac{W}{2g}k^2\omega^2$ to a body originally at rest.

Impacts cause impulsive forces of short duration. The total momentum of two colliding bodies remains constant and is the same after impact as before it. If there is imperfect elasticity in the deformation of solid bodies on impact, some heat is produced. But if the elasticity is perfect there is no loss of kinetic energy. It follows from the constancy of both momentum and kinetic energy that after impact of a small elastic body with a large stationary obstacle, the normal velocity of rebound is equal in magnitude to that of approach but reversed in direction, and no energy is given up or received. Also in the impact of a minute elastic body with a very large moving body the energy lost or gained can easily be determined. The minute body gains speed and kinetic energy by impact with an approaching large body and loses it by impact with a large receding body.

EXAMPLES III

1. A trolley bus weighs 3 tons and is running at 20 m.p.h. What is its kinetic energy in ft.-lb.? If the frictional resistances are constant and equal to 20 lb. per ton, how far will the bus run on the level before coming to rest and how far up an incline of 1 in 100?

2. A body weighing 40 kg. is moving at 10 metres per sec. Find its kinetic energy in joules.

3. When moving at 90 ft. per sec. a body has 10,000 ft.-lb. of kinetic energy. How much will it lose in decelerating to a speed of 20 ft. per sec.?

4. A hammer head weighing 15 lb. swings in part of a vertical circle. It is released at a point 3 ft. higher than the lowest point of the circle. Find the kinetic energy and the speed of the hammer head at the lowest point. If at that point it breaks a piece of metal and so parts with 30 ft.-lb. of energy, to what height will it rise on the other side?

5. A body weighing 6400 lb. has exerted upon it the following forces P lb. after moving x ft. from rest:

x ft. .	0	5	10	15	20	25	30	35	40
P lb.	700	690	680	650	640	630	640	620	610

If there is a constant resistance of 330 lb. to be overcome, find the speeds which the body will attain after moving 20 ft. and after 40 ft. from rest.

6. A press or punching machine is required to exert an average force of 3 tons through a distance of 0·5 in. If the energy is derived from that of two balls each weighing 40 lb. rotating in a circle, find the minimum linear speed at which the balls must be moving in order to supply the necessary energy, neglecting any loss.

7. A mass of 20 lb. oscillates under the action of a spring to which it is attached. The spring exerts a force of 30 lb. per in. of strain. If the mass is displaced 4 in. from its position of equilibrium find (*a*) its maximum speed after release, (*b*) its kinetic energy and its speed when 3 in. from its equilibrium position, and (*c*) the strain energy of the spring in this condition. Neglect the mass of the spring.

8. A lifting machine has a movement or velocity ratio of 6 to 1. A descending weight of 10 lb. used as an effort, just lifts a load of weight 40 lb. without acceleration. If the falling weight is increased to 15 lb., find its speed after descending 8 ft. and the tension in the rope to which it is attached. Neglect any inertia of the machine and assume that the energy lost in friction is unaffected by the addition of the extra 5 lb. to the effort.

9. Find the kinetic energy of a flywheel weighing a ton and having a radius of gyration of 3 ft. when making 240 r.p.m.

10. A flywheel weighing 2000 lb. has exerted upon it a torque of 500 lb.-ft. in excess of any frictional resistance. If its radius of gyration is 3 ft., find its speed in revolutions per minute after it has made 8 complete revolutions from rest.

11. A flywheel has a radius of gyration of 3·5 ft. and weighs 5000 lb. If it makes 500 revs. in coming to rest from a speed of 120 r.p.m., find how much work per revolution has been absorbed.

12. A flywheel is to run at 100 r.p.m. and is to give out 10,000 ft.-lb. energy with a reduction of speed to 98 r.p.m. If the radius of gyration is to be 4 ft., what should be the weight of the wheel?

13. The flywheel of a machine has 200,000 ft -lb. of kinetic energy at 240 r.p.m. How much will it give out in falling to 220 r.p.m.? If 80 per cent. of this is effectively used in overcoming a resistance through 2·5 in., find the average force exerted.

14. A flywheel, and the machine of which it forms a part, increase their kinetic energy by 10,000 ft.-lb. when the driving pulley speed increases from 100 to 101 r.p.m. How much kinetic energy have they when the pulley makes 100 r.p.m.?

15. An experimental flywheel weighs 212 lb. and is accelerated from rest by a load of 60 lb. attached to a cord wrapped round the spindle of the wheel. The effective radius of the spindle is 1 in. and the load descends 3 ft. in 5 sec. If 8 lb. of the load is required to overcome friction, find the radius gyration of the wheel.

CHAPTER IV

HYDRAULICS

37. Fluids. Matter may naturally be divided into three states in which it is well known: (1) Solid; (2) Liquid; (3) Gaseous (including vapours). Matter in the second and third of these states can flow and is therefore classed as *fluid.* The particles or molecules of a solid are in contact and adhere to one another; matter in this state has a degree of rigidity. It can offer a considerable resistance to change of shape when force is exerted upon it. The molecules of a liquid are in contact with one another but move freely one over another, so that liquids cannot maintain their shape, even under the gravitational force which is its own weight, without the help of a solid to contain it. While liquids such as water can be slightly reduced in bulk by high pressure, for most purposes we may regard them as incompressible. Gaseous matter differs from liquid in the fact that the molecules of a gas or a vapour do not remain in contact with other molecules but may be at considerable distances apart; they are in rapid motion, flying in all directions and colliding with one another and with the walls of any containing vessel. (We deal later with this in Art. 50.) A gas completely fills any space to which it has access, expanding if necessary to do so, while a liquid lies in the lower portion only, unless sufficient flows in to reach the upper part; in any case it does not expand to fill it but retains its own volume.

The mechanics of solids, liquids and gases have much in common and have, of course, common principles, but the foregoing fundamental differences in matters in the three states makes the applications of the principles somewhat different and there is some convenience in treating separately the mechanics of liquids (under the name of Hydraulics) and the mechanics of gases (sometimes called Pneumatics).

Although liquids change shape easily, they do possess some resistance to rapid change of shape and this property is called viscosity. This resistance is a tangential one opposing sliding motion or shearing: it is proportional to the speed of sliding motion of one molecule over another. At zero speed the resistance to sliding is zero; that is, the force between two molecules of a liquid at rest is entirely perpendicular to their surfaces in contact or wholly normal. Further, neither gases nor liquids can withstand tension so that the only kind of force which can be transmitted by a liquid or a gas at rest is a pressure perpendicular to the molecules. The frictional resistances between a liquid in motion and a solid body with which it is in contact is beyond the scope of our present work and we shall assume that the only force that can be transmitted between a liquid and the surface of a solid is a normal pressure; this is true of water at rest and very nearly true so long as it moves only at slow speeds.

38. Fluid Pressure. The intensity of pressure exerted within a fluid or on the walls of a containing vessel is measured by the force per unit of area, generally pounds per square inch or per square foot. This is very commonly called merely the *pressure*, although it is an intensity or degree of pressure and not the total force exerted which is also called the pressure, but to avoid confusion it is sometimes called the total pressure. It follows from the fact that only normal pressure can be transmitted by a stationary fluid that this normal pressure is equal in all directions at a given point in a fluid. And this point can be demonstrated experimentally by the use of pressure gauges.

39. Relation of Pressure to Depth in a Liquid. In a liquid large intensities of pressure may arise from the weight of the liquid when it extends continuously to a great depth. To investigate how much such pressure amounts to, imagine a vertical cylindrical shaped portion of a liquid at rest in a tank (Fig. 30), then, since the pressure on the curved vertical surface of the cylinder is everywhere horizontal it will have no vertical effect, and difference of the vertical pressure on the two flat ends must balance the weight of the cylindrical column of water. Putting this in the form of symbols, let A be the area of cross-section of the imagined cylinder, in square feet, h its height in feet, p_1 and p_2 the intensities of pressure at the top and bottom ends respectively in pounds-force per square foot. Then the volume of liquid in the cylinder is $A \times h$ cu. ft., and if the liquid weighs w lb. per cu. ft., the total weight of the cylinder of water is

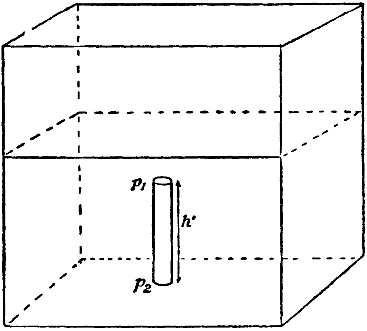

FIG. 30.—Variation of pressure with depth.

$$A \times h \text{ cu. ft.} \times w \text{ lb./cu. ft.}$$

or

$$A \,.\, h \,.\, w \text{ lb.}$$

and this must equal the net external force exerted upward on the cylinder which is

$$\begin{aligned} &A \text{ sq. ft.} \times p_2 \text{ lb.-force/sq. ft. (upward)} \\ &\qquad -A \text{ sq. ft.} \times p_1 \text{ lb.-force/sq. ft. (downward)} \\ &\qquad = A(p_2 - p_1) \text{ lb.-force (upward).} \end{aligned}$$

Hence, since this balances the downward force,

$$A(p_2 - p_1) \text{ lb.} = A \,.\, h \,.\, w \text{ lb.}$$

or

$$p_2 - p_1 = wh \text{ lb. per sq. ft.} \quad . \quad . \quad (1)$$

That is, the increase of pressure for a depth h ft. is **wh** lb. per sq. ft. **or** w lb. per sq. ft. per ft. of increase **of depth.**

If one end of the imaginary cylinder is in the free surface of the water the upward pressure of the surrounding water on the base of the cylinder is just equal to the weight of the cylinder of water.

Or, reckoning from the surface of the liquid, the pressure intensity at a depth h ft. will exceed the atmospheric pressure at this surface by

$$wh \text{ lb./sq. ft.}$$

And for pressures at great depths, the atmospheric pressure, equivalent to a depth of 34 ft. of water (see example below), becomes negligible.

The density of water (or its mass per unit volume) is about 62·4 lb. per cu. ft. at 60° F., so that for water $w = 62{\cdot}4$ lb.-force per cu. ft. and at a depth of 1 ft. the intensity of pressure (reckoned above atmosphere) is

$$p = 62{\cdot}4 \text{ lb./sq. ft.}$$

$$= \frac{62{\cdot}4}{144} \text{ or } 0{\cdot}433 \text{ lb./sq. in.}$$

and at a depth of h ft., pressure intensity

$$= 0{\cdot}433 \text{ lb./sq. in. per ft. depth} \times h \text{ ft.}$$

$$= 0{\cdot}433\ h \text{ lb./sq. in. (above the pressure of the atmosphere).}$$

Thus at a depth of 100 ft. the pressure would be $0{\cdot}433 \times 100 = 43{\cdot}3$ lb. per sq. in. above atmospheric pressure.

And to give a pressure of 1 lb. per sq. in. a depth or " head " of 1/0·433 or 2·31 ft. of water would be required, or $2{\cdot}31p$ ft. for p lb. per sq. in.

✓**Example.** What head of water would give a pressure equal to that of the atmosphere, namely, 14·7 lb. per sq. in.?

14·7 lb. per sq. in. $= 14{\cdot}7 \times 144$ lb. per sq. ft. $= 2117$ lb. per sq. ft. 1 cu. ft. of water weighs 62·4 lb. and a column h ft. high and 1 ft sq. in section would weigh

62·4*h* lb., which would give a pressure of 2117 lb. on the base provided that,

$$62{\cdot}4h = 2117$$

$$h = \frac{2117}{62{\cdot}4} = 34 \text{ ft. nearly.}$$

This is sometimes called the height of the "water-barometer." The atmospheric pressure corresponds to that of a column of mercury of which the height will be less in the ratio that mercury is heavier than water, viz. 13·6. Hence height of mercury column to give a pressure of 14·7 lb. per sq. in. is about about 34 ft./13·6 =2·5 ft. =30 in.

Manometer. For measuring moderate differences of pressure mercury in a U-tube is commonly used. The difference in height of the liquid on the two sides gives the difference of pressure in inches of mercury. If one end is open to the atmosphere, then the difference gives the pressure above that of the atmosphere.

40. Pressure on Submerged Surfaces. Since we know that the pressure per square foot at a depth *h* in a liquid which weighs *w* lb. per cu. ft. is *wh* lb. per sq. ft. (above atmospheric pressure), any horizontal surface such as the bottom of a tank of area A sq. ft. has upon it a total pressure of

$$wh \text{ lb./sq. ft.} \times \text{A sq. ft.} = wh\text{A lb. force} \quad . \quad . \quad (1)$$

But if we wish to know the total pressure on any surface which is not horizontal we have to deal with a pressure which is continuously varying across the surface, according to the varying depth, so we now consider the problem in symbols to find a simple rule.

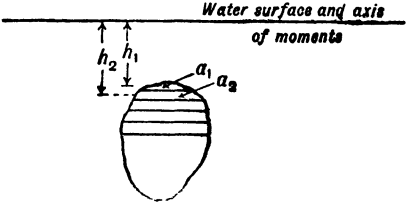

FIG. 31.—Pressure on submerged plate.

Take a vertical surface of total area A sq. ft. (shown in Fig. 31) and suppose it divided into a large number of parallel horizontal strips, the areas of which are a_1, a_2, a_3, etc., in square feet, and the depths of the strips are h_1, h_2, h_3, etc., ft. respectively below the surface of the liquid, then the total pressures on the successive strips will be

$$wh_1a_1,\ wh_2a_2,\ wh_3a_3,\ \text{etc.},$$

and the total pressure P, say, in pounds will be

$$\begin{aligned} P &= wh_1a_1 + wh_2a_2 + wh_3a_3 + \dots \text{ etc.} \\ &= w(h_1a_1 + h_2a_2 + h_3a_3 + \dots \text{ etc.}) \quad . \quad . \quad (2) \end{aligned}$$

Now the sum of the products h_1a_1, h_2a_2, etc., constitute what is called the moment of the area A about an axis which is the line of intersection of the vertical plane of the submerged surface with the surface of the liquid, and this moment is such that

$$h_1a_1 + h_2a_2 + h_3a_3 + \text{ etc.} = \bar{h} \,.\, A \quad . \quad . \quad . \quad (3)$$

where $\bar{h}$ is the distance of the centroid, centre of area or so-called centre of gravity of the total area A from the same axis. Hence we may write the equation (2) as

$$P = w \,.\, \bar{h} \,.\, A \text{ lb.} \quad . \quad . \quad . \quad . \quad (4)$$

where $\bar{h}$ is the depth of the centre of gravity of the surface.

To put the matter in another way, the pressure per square foot $\bar{p}$, say, at a depth $\bar{h}$ will be $w\bar{h}$, so that

$$P = \bar{p}A \text{ lb.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where $\bar{p}$ is the mean pressure intensity or the intensity of pressure at the depth of the centre of gravity or centroid of the immersed surface. In words,

Total pressure in lb. = area in sq. ft.
× pressure per sq. ft. at c.g. of area.

If a surface is neither horizontal nor vertical but oblique at some intermediate inclination, the same rule applies, where $\bar{h}$ still refers to the *vertical* depth of the centre of gravity.

✓ **Example 1.** Find the total pressure on a lock gate the width of which is 20 ft., when the depth of fresh water is 15 ft.

Depth of centre of gravity of wetted area

$$=15/2 \text{ ft.} = 7{\cdot}5 \text{ ft.}$$

Pressure at depth of centre of gravity

$$=7{\cdot}5 \text{ ft.} \times 62{\cdot}4 \text{ lb./cu. ft.} = 468 \text{ lb./sq. ft.}$$

$$\text{Total wetted area} = 20 \text{ ft.} \times 15 \text{ ft.} = 300 \text{ sq. ft.}$$

$$\text{Total pressure} = 468 \text{ lb./sq. ft.} \times 300 \text{ sq. ft.}$$

$$= 140{,}400 \text{ lb}$$

Example 2. A submerged rectangular sluice gate is 3 ft. by 2 ft., having its long sides vertical and short sides horizontal. The top side is 5 ft. below the surface of water. Find the total water pressure on the gate. Take salt water at 64 lb. per cu. ft.

The depth of the centre of gravity below the top side

$$=3/2 \text{ ft.} = 1{\cdot}5 \text{ ft.}$$

Depth of centre of gravity below surface of water

$$=5 + 1{\cdot}5 = 6{\cdot}5 \text{ ft.}$$

Pressure at depth of centre of gravity

$$=6{\cdot}5 \text{ ft.} \times 64 \text{ lb./cu. ft.} = 426 \text{ lb./sq. ft.}$$

$$\text{Total area} = 3 \text{ ft.} \times 2 \text{ ft.} = 6 \text{ sq. ft.}$$

$$\text{Total pressure} = 426 \text{ lb./sq. ft.} \times 6 \text{ sq. ft.} = 2556 \text{ lb.}$$

Example 3. The tank shown in elevation in Fig. 32 is 6 ft. broad (in a direction perpendicular to the diagram).

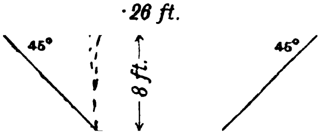

FIG. 32.—Problem on tank.

Find the pressure on (*a*) a sloping side, (*b*) a vertical side, and (*c*) on the bottom when the tank is full of water.

(*a*) Length of sloping side

$=8\sqrt{2}$ ft. $=8\times1{\cdot}414=11{\cdot}31$ ft.

Area of sloping side

$=11{\cdot}31$ ft. $\times 6$ ft. $=67{\cdot}86$ sq. ft.

Depth of centre of gravity of sloping side

$=8/2=4$ ft.

Pressure on sloping side

$=67{\cdot}86$ sq. ft. $\times 4$ ft. $\times 62{\cdot}4$ lb./cu. ft.

$=16{,}960$ lb.

(*b*) We can divide the vertical side into a central rectangle 8 ft. deep and $26-2\times8=10$ ft. long, and two right-angled triangles 8 ft deep and each having a base in the water surface 8 ft. long. The depth of the centres of gravity of these triangles will be 8/3 ft. below the water surface.

Rectangular area $=10$ ft $\times 8$ ft. $=80$ sq. ft.

Depth of centre of gravity $=8/2$ ft. $=4$ ft.

Total pressure on this part

$=80$ sq. ft. $\times 4$ ft. $\times 62{\cdot}4$ lb./cu. ft.

$=19{,}970$ lb.

Two triangular areas $=2\times8\times8\times\frac{1}{2}=64$ sq. ft.

Depth of centre of gravity $=8/3$ ft. $=2{\cdot}67$ ft.

Total pressure on those parts

$=64$ sq. ft. $\times 2{\cdot}67$ ft. $\times 62{\cdot}4$ lb./cu. ft

$=10{,}650$ lb.

Total pressure on vertical side

$=(19{,}970+10{,}650)$ lb.

$=30{,}620$ lb.

(*c*) Area of bottom $=10$ ft. $\times 6$ ft. $=60$ sq. ft.

Depth of centre of gravity $=8$ ft.

Total pressure $=60$ sq. ft. $\times 8$ ft. $\times 62{\cdot}4$ lb./cu. ft.

$=29{,}950$ lb.

41. Transmission of Pressure and Energy. We now return to the equality of pressure in all parts of a body of fluid except in so far as there is some variation with depth, due to the weight of the fluid, particularly with a heavy liquid. But in hydraulic machinery the pressure to which the water or other fluid (such as oil) is subjected is often so high that we can neglect any differences due to slight differences in depth at different parts of any containing vessel, and we shall do so in what follows.

The equality of intensity of pressure makes water or other liquids under pressure a very convenient medium for transferring the force exerted in one place to another place where the use of force is required. And it also allows a small force exerted through a large distance to overcome a large resistance through a smaller distance as an alternative to using some form of gearing such as is used in the various lifting machines studied in an earlier course.

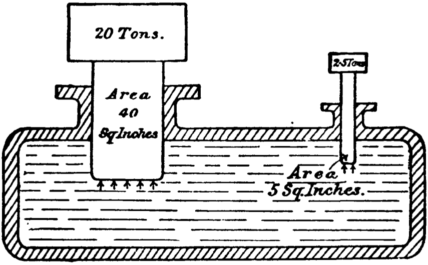

Fig. 33.—Transmission of pressure by liquid.

The principle will be most easily understood from an ideal case diagrammatically shown in Fig. 33. This represents two frictionless water-tight pistons or plungers subject to the pressure of the water contained in a vessel capable of holding it under considerable pressure. If the smaller plunger has an area of cross-section of 5 sq. in. and carries a load of 2·5 tons, the total force divided by this area, or 2·5 tons/5 sq. in., gives a pressure of 0·5 ton per square inch which the plunger exerts downward on the water, and as a reaction the water exerts a total force of 2·5 tons at 0·5 ton per square inch upward on the plunger. This pressure of

0·5 ton per square inch will also be exerted on the larger plunger and if its area of cross-section is, say, 40 sq. in., the total load it can lift (or other resistance it can overcome) is 0·5 ton per square inch × 40 sq. in. = 20 tons.

If friction be neglected then, with such an arrangement a force of 2·5 tons can lift a load of 20 tons, that is, 8 times as much as the "effort" of 2·5 tons, because the larger plunger has an area of cross-section 40/5 or 8 times as large as the small one. Since the volume of water displaced by the smaller plunger must be equal to that previously occupied by the larger one before it was raised (for the total volume of water does not change in the operation) the vertical movement of the small plunger must be 8 times as great as that of the large one. For example, if the small plunger comes down 10 in., it displaces 10 in. × 5 sq. in. = 50 cu. in. of water, and to accommodate this water the larger plunger must vacate 50 cu. in. of space. And since its area of cross-section is 40 sq. in. it must move up by 50 cu. in./40 sq. in. = 1·25 in., which is one-eighth of the motion of the small plunger. Thus, regarded as a lifting apparatus, the velocity ratio or movement ratio is 8 and the mechanical advantage 20 tons/2·5 tons is also 8, friction being assumed to be zero.

Put in symbols, let W_1 be the effort in pounds, A_1 sq. ft. the area of cross-section of the effort plunger, W_2 the load in pounds, and A_2 sq. ft. the area of cross-section of the load plunger. Then if the load moves up l_2 ft. and the effort moves downwards l_1 ft., since the volumes must be equal,

$$(A_2 \times l_2) \text{ cu. ft.} = A_1 \times l_1 \text{ cu. ft.} \quad \ldots \quad (1)$$

or

$$\frac{l_1}{l_2} = \frac{A_2}{A_1} \quad \ldots\ldots \quad (2)$$

which is the velocity ratio.

Fig. 33 illustrates the principle of the hydraulic press and the hydraulic lifting jack, in both of which the effort is often applied by a lever to a small plunger or ram, and acting through a large distance, can lift a large load.

But we must learn, too, from Fig. 33, a very important relation in regard to the energy transferred. The load

2·5 tons loses potential energy by coming to a lower level and the load of 20 tons gains an equal amount of potential energy by being lifted and the work done by the effort and that done on the load is a measure of the transfer of energy from one to the other. And quantitatively this amount of energy can be stated in terms of the number of units of work done by one plunger and on the other. Let us put this down in symbols. Using those already employed above, let p be the intensity of pressure in pounds per square *foot.*

$$\text{Then } p=\frac{\text{total force}}{\text{area}}=\frac{W_2 \text{ lb.}}{A_2 \text{ sq. ft.}}=\frac{W_1 \text{ lb.}}{A_1 \text{ sq. ft.}} \quad . \quad (3)$$

For axial movements l_2 and l_1 feet,

Work done on the water

$$=W_1 l_1=p \text{ lb./sq ft.} \times A_1 \text{ sq. ft.} \times l_1 \text{ ft.}$$
$$=pA_1 l_1 \text{ ft.-lb.} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

and this is equal to,

Work done by the water

$$=W_2 l_2=p \text{ lb./sq. ft.} \times A_2 \text{ sq. ft.} \times l_2 \text{ ft.}$$
$$=pA_2 l_2 \text{ ft.-lb.} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

In these two equal expressions for the work done, the parts $A_1 l_1$ and $A_2 l_2$ both represent the volume in cubic feet of water displaced. Let this be represented by V (cu. ft.), then the work done may be written

$$\text{Work}=pV \text{ ft.-lb.} \quad . \; . \; . \; . \; . \; . \; . \quad (6)$$
$$=\text{pressure per } \textit{square foot} \times \text{volume in } \textit{cubic feet} \quad . \; . \quad (7)$$

This expression for work done is important. It holds good for any fluid entering at constant pressure p a space of volume V whatever the shape of that space. It cannot enter a space at constant pressure unless it is overcoming a resistance driving something in front of it and doing work in making room for itself and the product pV is the measure of that work done. Note particularly that the product of p in pounds per square *foot* and V in cubic *feet* gives work in foot-pounds. Similarly, p in pounds per square *inch* and V in cubic *inches* would give work in inch-pounds. There is great liability to numerical error here unless consistent units are used.

The expression pV for work done by a fluid at constant pressure p applies equally to a liquid and a gas and we shall employ it in connection with heat engines.

It has another form which is often useful for 1 lb. of fluid. If the fluid weighs w lb. per cu. ft., the volume of 1 lb. is $\frac{1}{w}$ cu. ft. Hence the work done by 1 lb. of a fluid against constant resistance at a pressure p lb. per sq. ft. is given by

$$\text{Work per lb.} = p \times \text{volume per lb.} = \frac{p}{w} \text{ ft.-lb.} \qquad (8)$$

and this represents the energy transmitted per pound of water moved, a form which we shall use later (Art. 45).

Note that for the transmission of the energy movement is essential. The water in fact transmits the energy in much the same way as a solid rod, such as a piston rod, at a rate of $\frac{p}{w}$ ft.-lb. per pound of water which crosses any transverse section along the path of transmission, i.e. between the two plungers in Fig. 33.

✓**Example 1.** A hydraulic press is operated by an effort of 40 lb. exerted at right angles to a lever and 30 in. from its fulcrum. The plunger is pushed also at right angles to the lever and 2 in. from the fulcrum; it is 1 in. in diameter. The ram or large plunger is 10 in. in diameter. Find the force which can be exerted by the press if the efficiency of the apparatus at this load is 85 per cent., i.e. if 15 per cent. of the energy is wasted in friction.

Firstly neglecting frictional losses, from the principle of moments,

$$40 \text{ lb.} \times 30 \text{ in.} = \text{force on plunger} \times 2 \text{ in.}$$

$$\text{Force exerted on plunger by lever} = 40 \text{ lb.} \times \frac{30 \text{ in.}}{2 \text{ in.}}$$
$$= 600 \text{ lb.}$$

$$\text{Area of plunger} = 0{\cdot}7854 \times 1 \text{ sq. in.} = 0{\cdot}7854 \text{ sq. in.}$$

$$\text{Pressure intensity in water} = \frac{600 \text{ lb.}}{0{\cdot}7854 \text{ sq. in.}}$$
$$= 764 \text{ lb./sq. in.}$$

$$\text{Area of ram} = 0{\cdot}7854 \times 10^2 = 78{\cdot}54 \text{ sq. in.}$$

Total pressure on ram

$$= 764 \text{ lb./sq. in.} \times 78{\cdot}54 \text{ sq. in.} = 60{,}000 \text{ lb.}$$

Allowing for 15 per cent. loss,

$$\text{Total pressure} = 60{,}000 \text{ lb.} \times 0{\cdot}85 = 51{,}000 \text{ lb.}$$

Example 2. In a hydraulic jack the leverage of the handle is 15 to 1, what must be the ratio of the diameter of the large plunger or ram to that of the small plunger worked by the handle if a force of 20 lb. on the handle is to lift a load of 2000 lb., assuming that the efficiency is 70 per cent.

$$\text{Force on plunger} = 15 \times 20 \text{ lb.} = 300 \text{ lb.}$$

$$\text{Pressure in water } p = \frac{300 \text{ lb.}}{0{\cdot}7854 d^2} \text{ lb. per sq. in.}$$

where d = diam. of plunger in inches,

This will give an effective force for lifting of 70 per cent. of $p \times 0{\cdot}7854 D^2$, where D = diam. of ram in inches, and this must be equal to 2000 lb. Hence,

$$0{\cdot}7 \times \frac{300 \text{ lb.}}{0{\cdot}7854 d^2} \times \frac{0{\cdot}7854 D^2}{1} = 2000 \text{ lb.}$$

$$\frac{D^2}{d^2} = \frac{2000}{0{\cdot}7 \times 300} = \frac{20}{2{\cdot}1} = 9{\cdot}52$$

$$\frac{D}{d} = \sqrt{9{\cdot}52} = 3{\cdot}1$$

Example 3. Hydraulic power is supplied from pipes in which the water is at 500 lb. per sq. in. pressure.

How many gallons per hour will be required to operate a motor or machine working at 20 h.p. if the efficiency is 70 per cent.?

The work to be done in 1 hr.

$$=20 \times 33{,}000 \text{ ft.-lb.} \times 60 = 39{,}600{,}000 \text{ ft.-lb.}$$

Work obtainable from V cu. ft.

$$=\frac{70}{100} \times \frac{500 \text{ lb.}}{\text{sq. in.}} \times \frac{144 \text{ sq. in.}}{\text{sq. ft.}} \times V \text{ cu. ft.}$$

$$=\frac{70}{100} \times 500 \times 144V \text{ ft.-lb.}$$

and this must give 39,600,000 ft.-lb., hence

$$V=\frac{39{,}600{,}000 \times 100}{70 \times 500 \times 144}=786 \text{ cu. ft.}$$

and at 6·24 gallons to 1 cu. ft., the quantity required is

$$786 \text{ cu. ft.} \times 6{\cdot}24 \text{ gal. per cu. ft.} = 4734 \text{ gallons.}$$

42. Suction or Lift Pump. This common piece of hydraulic apparatus illustrates some of the principles of this chapter. Consider first what suction means. If a tube is placed with one end in a bowl of water and a human mouth draws air out of the top end of the tube, the pressure in the tube falls below that of the atmosphere: this is called forming a "partial vacuum." But the water in the bowl has exerted upon it the pressure of the atmosphere which rests on its surface. This pressure, being now greater than that in the upper part of the tube, drives the water up the tube until the weight of water standing in the tube just balances the difference between the upward pressure (atmospheric) of the water at the bottom of the tube and the reduced air pressure in the upper part of the tube. All "suction" is of this nature, that is, the pressing of a liquid forward by a full atmospheric pressure acting against a reduced pressure; there is no appreciable pulling of a liquid. In other words, there is practically no tension possible in a fluid.

A suction pump is shown diagrammatically in Fig. 34

and it acts in much the same way as mouth suction. It consists of a cylindrical pump barrel B, in which an air-tight plunger or bucket A, is driven up and down by hand or other agency. This bucket has one or more valves C which can only open upwards to allow the water to pass through them from the lower to the upper side. A pipe F attached to the bottom of the pump barrel B leads down to the water in the well, the lower end of F being always submerged. At the top of the pipe F is a valve E which opens upwards to allow ascending water to pass into the pump barrel B.

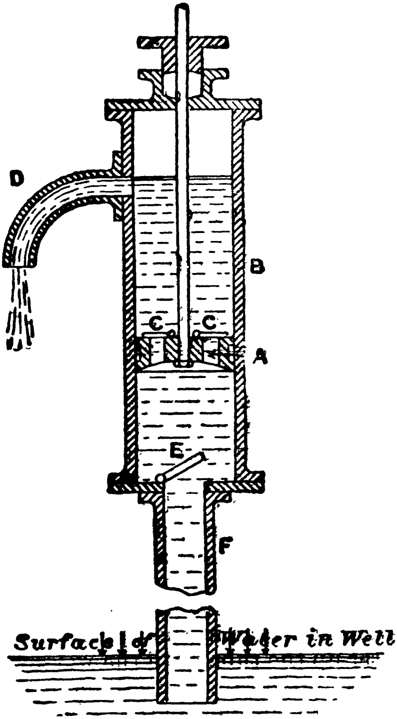

Fig. 34.—Suction pump.

When the pump is started, B and F will be full of air, and water will not be delivered till this is cleared away. Suppose the bucket A is at the top of its stroke. As it is moved downwards, E is closed, and the valves C open and so allow the air in the lower part of the barrel to pass through the bucket. On the upward stroke of the bucket the valves C close and above E the pressure is lowered below that of the atmosphere; the valve E opens and air is drawn out of F with the result that the pressure in F also is lowered below that of the atmosphere. But the water in the well has the pressure of the atmosphere pressing down on its surface and transmitted normally in all directions in the water. Consequently some water is driven up the pipe F by this pressure. The amount will be such that the water, together with the (reduced) air pressure above it, exerts a downward force equal to the upward force exerted by the water in the well due to the pressure of the atmosphere. This process is repeated with successive downward and upward strokes till the space below the bucket is full of water instead of air, and then water passes through the valves E and C on the upward and downward strokes respectively. After this, water which has passed through the valves C on a downward stroke is lifted directly by the bucket on its upward stroke, this final lift not being by suction (or atmospheric pressure) but by direct lift. The water, after this direct lift, flows out of a spout D.

Since the water is driven up the pipe by the pressure of the atmosphere, or the excess of it over that in the pump barrel, the height to which water can be thus lifted is limited by this available pressure.

We have seen in the example in Art. 39 that the limit of height for 14·7 lb. per sq. in. pressure of the atmosphere is 34 ft. This height of lift could only be achieved if we could reduce the pressure at the top of the lift pipe to zero. For various reasons, particularly because we cannot make the joints, valves, etc., perfectly air-tight, we can only get a moderately low pressure by such an apparatus and the direct suction lift in practice seldom exceeds 26 ft. There is no corresponding limit to the remaining lift, that is, to the distance from the bottom of the pump barrel at E to the spout at D. But for practical and mechanical reasons this lift is usually short.

Volume Delivered. If the area of cross-section of the barrel and plunger or bucket is A sq. in., say, and the stroke is l in., the volume swept through at each stroke is $A \times l$ cu. in., and this would be the nominal delivery on each *upward* stroke. But actually, apart from leakages after valves are closed, some water slips back through the valves while they are closing, for valves cannot act instantaneously, particularly when, as shown, they act automatically with changes of pressure. The actual delivery is therefore less than the nominal by the amount of this "slip."

Example 1. The barrel of a suction pump is 5 in. diameter and the stroke is 8 in. How many upward strokes of the bucket will be required to lift 1000 galls. of water if there is 12 per cent. of slip?

Volume lifted per stroke

$$=\frac{88}{100} \text{ of area of section} \times \text{length of stroke}$$

$$=\frac{88}{100} \times \frac{0 \cdot 7854 \times 5^2}{144} \text{ sq. ft.} \times \frac{8}{12} \text{ ft.}$$

$$=0 \cdot 0714 \text{ cu. ft.}$$

Volume to be lifted

$$=\frac{1000 \text{ gal.}}{6 \cdot 24 \text{ gal./cu. ft.}}=160 \cdot 3 \text{ cu. ft.}$$

Number of strokes required

$$=\frac{160 \cdot 3}{0 \cdot 0714}=2245.$$

Example 2. If the pump in Example 1 delivers water 80 ft. above the bucket, find the total pull in the rod which lifts the bucket, in addition to that from its own weight. (Neglect any acceleration of the water and any partial vacuum under the bucket.)

Volume of water above the bucket

$$=\text{area} \times \text{length of column}$$

$$=\frac{0 \cdot 7854 \times 25}{144} \text{ sq. ft.} \times 80 \text{ ft.}$$

$$=10 \cdot 91 \text{ cu. ft.}$$

Tension in rod $=$ weight of water

$$=10 \cdot 91 \text{ cu. ft.} \times 62 \cdot 4 \text{ lb./cu. ft.}$$

$$=681 \text{ lb.}$$

43. Force Pump. This differs from the lift pump in having no valves in the plunger. It is shown diagrammatically in Fig. 35. The plunger A is solid and on its upward stroke it draws water into the barrel B through the suction valve C, just as in the previous case, i.e. by the pressure of the atmosphere. But usually this suction lift is short, not approaching the full lift of 34 ft. corresponding to the full atmospheric pressure. During this upward stroke the delivery valve D is kept closed by the pressure of the column of water in the delivery pipe E. On the return or downward stroke of the plunger, the pressure of the water above it closes the suction valve C and the pressure in the barrel becomes greater than that of the water in E and opens the delivery valve D, through which the water is forced up the delivery pipe E. As in the previous type there is, of course, some "slip" of water at the valves. The greater part of the lift is usually effected by the downward thrust of the plunger. Thus it is not limited by the magnitude of the atmospheric pressure.

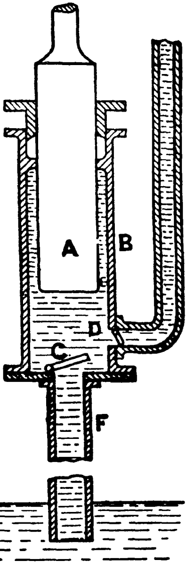

FIG. 35.—Force pump.

Example 1. A force pump delivers water against a head of 180 ft. What average force will be required on the pump plunger during the delivery stroke if the mechanical efficiency is 85 per cent., i.e. if 15 per cent. of the energy supplied is wasted in friction?

Pressure due to 180 ft. head of water

$$=180 \text{ ft.} \times 62{\cdot}4 \text{ lb./cu. ft.}$$

$$=11{,}230 \text{ lb. per sq. ft.}$$

$$\text{Area of plunger} = \frac{0{\cdot}7854 \times 5^2}{144} \text{ sq. ft.}$$
$$= 0{\cdot}1363 \text{ sq. ft.}$$

Force exerted by plunger on water

$$= 0{\cdot}1363 \text{ sq. ft.} \times 11{,}230 \text{ lb./sq. ft.}$$
$$= 1530 \text{ lb.}$$

Driving force exerted on plunger

$$= \tfrac{100}{85} \times 1530 = 1800 \text{ lb.}$$

Example 2. Find the horse-power required for the pump in Example 1, neglecting the work done in the suction stroke if the pump has a stroke of 8 in. and makes 45 working strokes per minute.

Work per stroke = force exerted on plunger × stroke

$$= 1800 \text{ lb.} \times \tfrac{8}{12} \text{ ft.} = 1200 \text{ ft.-lb.}$$

Work done per minute

$$= 1200 \text{ ft.-lb.} \times 45 = 54{,}000 \text{ ft.-lb.}$$

$$\text{power required} = \frac{54{,}000 \text{ ft.-lb./min.}}{33{,}000 \text{ ft.-lb./min.}}$$
$$= 1{\cdot}64 \text{ h.p.}$$

Example 3. If the slip in the pump in Examples 1 and 2 is 10 per cent., how many gallons of water will be delivered per minute ?

Volume displaced by plunger = area × stroke

$$= 0{\cdot}1363 \text{ sq. ft.} \times \tfrac{8}{12} \text{ ft.}$$
$$= 0{\cdot}0909 \text{ cu. ft.}$$

$$\text{Water delivered per stroke} = \tfrac{90}{100} \times 0{\cdot}0909 \text{ cu. ft.}$$
$$= 0{\cdot}0818 \text{ cu. ft.}$$

$$\text{Water delivered per minute} = 0{\cdot}0818 \times 45 = 3{\cdot}68 \text{ cu. ft.}$$

$$\text{Gallons per minute} = 3{\cdot}68 \times 6{\cdot}24 = 23 \text{ gallons.}$$

44. Water in Motion. So far in our study of the mechanics of liquids we have considered liquids at rest or in slow motion when transmitting energy as well as force. But when in motion, because it has mass, a liquid carries in itself kinetic energy which, though negligible at very low speeds, becomes important when the motion is rapid, since it is proportional to the square of the speed. The study of the mechanics of water in motion is mostly appropriate to a higher stage of work than that of this book. But a few applications of mechanics of motion to liquids are important and form excellent illustrations of the principles of mechanics. And they may be treated very simply if we limit ourselves of cases where friction and viscosity are neglected. In many problems relating to the flow of water, friction is not negligible, but we shall not begin the study of these cases at this stage.

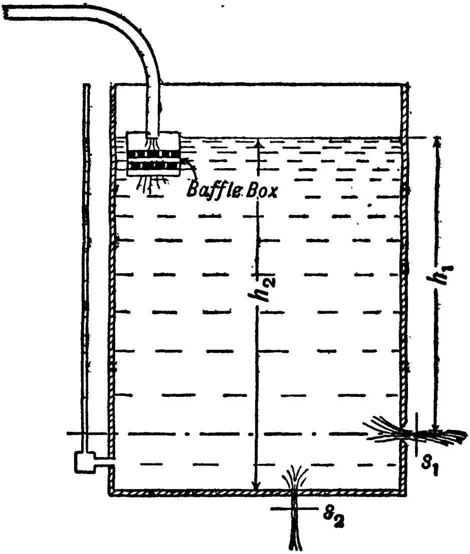

FIG. 36.—Flow from orifices.

Flow from an Orifice. We may first consider the flow of water through a hole or orifice in a vessel containing water under constant pressure. Fig. 36 represents an experimental tank in which the outflow is constantly replenished by an inflow so arranged as to give a large volume of nearly still water maintained at a constant level. Experimental orifices of various shapes may be used, but a circular one is commonest; they are usually sharp edged, as shown. This reduces the friction of flow and gives a clear stream with a fairly definite boundary instead of a broken stream of which it would be impossible to measure the diameter. The stream, on issuing from the orifice, contracts to a smaller cross-section than that of the orifice and the ratio which the minimum sectional area at S_1 or S_2 bears to the full area of the orifice is called the coefficient of contraction of the orifice. (It can be measured with some difficulty.) If the level of the still surface in the tank is h_1 or h_2 ft. above the centre of an orifice (Fig. 36), the velocity of outflow is nearly equal to that of a body falling freely through a height h_1 or h_2 ft., viz. $\sqrt{2gh_1}$ or $\sqrt{2gh_2}$ ft./sec. (see (1), Art. 10). The ratio which the actual velocity v at S_1, say, bears to the velocity $\sqrt{2gh_1}$ ft./sec. due to falling freely h_1 ft. is called the coefficient of velocity, which is generally about 0·97. The volume V in cubic feet flowing per second would be that of a cylinder having a cross-sectional area a_1 sq. ft. equal to that of the stream at S_1 and a length v ft. or

$$V = a_1 \times v \text{ or about } 0{\cdot}97 a_1 \sqrt{2gh_1} \text{ cu. ft./sec.}$$

And if a = area of orifice in square feet the coefficient of contraction is a_1/a, so that the volume flowing may be written

$$V = a \times \frac{a_1}{a} \times v$$

$$= a \times \text{coefficient of contraction} \times \text{coefficient of velocity} \times \sqrt{2gh_1} \quad . \quad (1)$$

and the product of these two coefficients is called the coefficient of discharge, so that

$$V = a \times \sqrt{2gh_1} \times \text{coefficient of discharge}$$

or

$$V = ka\sqrt{2gh_1} \text{ cu. ft./sec.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where k is the coefficient of discharge. This coefficient of discharge k, which is about 0·62 for a circular sharp-edged orifice, is determined by measuring the volume V and the linear dimensions a and h_1. With a coefficient of contraction of, say, 0·64 we have

Coefficient of discharge
=coefficient of contraction × coefficient of velocity (3)

e.g. $$0{\cdot}62 = 0{\cdot}64 \times 0{\cdot}97.$$

The coefficient of discharge is easily determined from V and the coefficient of contraction can be measured by special apparatus. The coefficient of velocity can be found by calculation from the above relation.

The fact that with negligible friction the velocity of outflow would be equal to that of falling freely through a height h_1 ft. could be deduced from the principle of the conservation of energy. For since the incoming water is at a height h_1 above the orifice, it has a potential energy of h_1 ft.-lb. per lb. above that at outflow. And if there is no loss this must be the kinetic energy per pound at outflow, which is $\frac{v^2}{2g}$ (see (1 Art. 31, when W=1). Hence

$$\frac{v^2}{2g} = h_1$$

or $$v = \sqrt{2gh_1} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Velocity from Pressure. But we might usefully look at the matter in another way. At a depth h_1 the pressure on a square foot is wh_1 lb., where w is the weight of a cubic foot of the liquid, or

$$p = wh_1 \text{ (as in Art. 39)}$$

and this pressure does work on the outflowing stream, which is $p \times$ volume. And if we consider 1 lb. of water, its volume is $\frac{1}{w}$ cu. ft., so that the work done on it is $\frac{p}{w}$ ft.-lb., which must be equal to the kinetic energy or

$$\frac{p}{w} = \frac{v^2}{2g} \quad \text{or} \quad v = \sqrt{\frac{2gp}{w}} \text{ ft./sec.} \quad . \quad . \quad . \quad (5)$$

and since $p=wh_1$, hence

$$h_1=\frac{v^2}{2g} \quad \text{or} \quad v=\sqrt{2gh_1}\ \text{ft./sec.} \quad . \quad . \quad . \quad . \qquad (6)$$

The relation (5) would give us the velocity of flow from an orifice in a vessel containing water at pressure p lb. per sq. ft., whether due to a depth of water h_1 ft., or to the force on a pump plunger, or any other cause.

✓ **Example 1.** Find the diameter of a sharp-edged circular orifice to discharge 2400 gal. per hour under a head of 40 ft. of water.

$$\begin{aligned}\text{Discharge} &= 2400\ \text{gal./hr.} \times (1/6{\cdot}24)\ \text{cu. ft./gal.}\\ &= 385\ \text{cu. ft./hr.}\\ &= 0{\cdot}107\ \text{cu. ft./sec.}\end{aligned}$$

If a is the area of orifice in square feet, then, assuming a coefficient of discharge of 0·62, the discharge in cubic feet per second under a head of 40 ft. is

$$0{\cdot}62 \times a \times \sqrt{2 \times 32{\cdot}2 \times 40} = 0{\cdot}107\ \text{cu. ft./sec.}$$

$$0{\cdot}62 \times a \times 50{\cdot}8\ \text{ft./sec.} = 0{\cdot}107\ \text{cu. ft./sec.}$$

$$\begin{aligned}a &= \frac{0{\cdot}107\ \text{cu. ft./sec.}}{0{\cdot}62 \times 50{\cdot}8\ \text{ft./sec.}}\\ &= 0{\cdot}00275\ \text{sq. ft.}\\ &= 0{\cdot}3961\ \text{sq. in.}\end{aligned}$$

And if d is the diameter in inches,

$$d^2 = \frac{0{\cdot}3961}{0{\cdot}7854} = 0{\cdot}504\ \text{in.}^2$$

$$d = \sqrt{0{\cdot}504} = 0{\cdot}71\ \text{in.}$$

✓ **Example 2.** A jet 1 in. diameter issues from a nozzle connected to a supply of water at a pressure of 200 lb. per sq. in. Find the velocity of the jet, the flow in cubic feet per minute, and the horse-power available from the jet if 2 per cent. of the energy is wasted in friction.

Work done in foot-pounds by the water supply on the water issuing from the jet = pressure per square foot

×volume in cubic feet. Considering the quantities per pound issuing,

$$\text{Volume} = \frac{1}{w} = \frac{1\text{ lb.}}{62{\cdot}4\text{ lb./cu. ft.}} = \frac{1}{62{\cdot}4}\text{ cu. ft.}$$

Work done

$$= \text{pressure/sq. ft.} \times \text{volume in cubic feet per lb.}$$

$$= 200\text{ lb./sq. in.} \times 144\text{ sq. in./sq. ft.} \times \frac{1}{62{\cdot}4}\text{ cu. ft./lb.}$$

$$= \frac{200 \times 144}{62{\cdot}4}\text{ ft.-lb./lb.}$$

$$= 461{\cdot}5\text{ ft.-lb./lb.}$$

Loss of 2 per cent (deduct) $= 9{\cdot}2$ ft.-lb./lb.

Kinetic energy in jet $= 452{\cdot}3$ ft.-lb./lb.

$$\frac{v^2}{64{\cdot}4\text{ ft./sec.}^2} = 452\text{ ft.-lb./lb.}$$

$$v^2 = 64{\cdot}4 \times 452\text{ (ft./sec.)}^2$$

$$v = \sqrt{64{\cdot}4 \times 452}\text{ ft./sec.}$$

$$= 172\text{ ft./sec.}$$

Flow in cubic feet per minute

$$= 172\text{ ft./sec.} \times 60\text{ sec./min.} \times \frac{0{\cdot}7854}{144}\text{ sq. ft.}$$

$$= \frac{172 \times 60 \times 0{\cdot}7854}{144}\text{ cu.ft./min.}$$

$$= 56{\cdot}3\text{ cu. ft./min.}$$

Since each pound has 452 ft.-lb. of K.E.,

Kinetic energy available per minute from jet

$$= 56{\cdot}3\text{ cu.ft./min.} \times 62{\cdot}4\text{ lb./cu. ft.} \times 452\text{ ft.-lb./lb.}$$

$$= 56{\cdot}3 \times 62{\cdot}4 \times 452\text{ ft.-lb./min.}$$

$$= 1{,}588{,}000\text{ ft.-lb./min.}$$

$$\text{h.p.} = \frac{1{,}588{,}000}{33{,}000} = 48{\cdot}0.$$

45. Head and Energy in Flow. We have seen that water moving, even slowly, under pressure transmits not only pressure but energy (Art. 41). In this respect it acts like solid material, say a steel rod such as the piston rod or the connecting rod of an engine, except that it can transmit only a thrust and not a pull. In both cases the energy transmitted in any given time is the intensity of pressure multiplied by the volume swept through, say, p lb. per sq. ft. $\times V$ cu. ft. $= pV$ ft.-lb. Or per pound of water (or steel) passing a given plane perpendicular to the motion, the energy transmitted is

$$p \times \text{volume of 1 lb. in cu. ft.} = \frac{p}{w} \quad . \quad . \quad (1)$$

where w is the weight in pounds of 1 cu. ft. or $\frac{1}{w}$ the volume in cu. ft. of 1 lb.

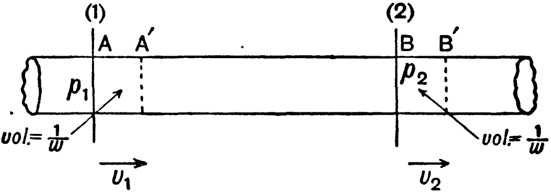

FIG. 37.—Transmission of pressure and energy.

It is sometimes said that water under pressure thereby has energy $\frac{p}{w}$ per lb., but this energy is no more contained in the water than is the energy transmitted by a piston (or other) rod, contained in the rod while transmission takes place. And water at rest under pressure does not contain energy $\frac{p}{w}$ per lb. any more than does, say, steel under pressure in a structure.[1]

Kinetic energy at $v^2/2g$ ft.-lb. per lb. may more properly be said to be in water moving at v ft. per sec. To emphasise a point, so often misstated, consider a pipe of uniform bore (Fig. 37) with water flowing at a uniform speed $v_1 = v_2$ ft. per sec. and at a pressure $p_1 = p_2$ lb. per sq. ft. While 1 lb. of water flows past a section (1), an equal volume $\left(\frac{1}{w}\right)$ cu. ft. flows past section (2). The work done *on* the water AB by the water to the left of A is $p \times$vol. of 1 lb.$=\frac{p}{w}$ ft.-lb., and the work done by the water AB on that to the right of B is also $\frac{p}{w}$. This work is transmitted in the same way along any length from a supply of mechanical energy to a place where it is used to do work, with or without conversion, to some other form of energy. And if the pipe is horizontal, the total energy passing per pound of water *flowing* is

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[1] Both water and steel contain a *small* amount of elastic strain energy when under pressure, but this is negligible in comparison with p/w per lb. at ordinary pressures.

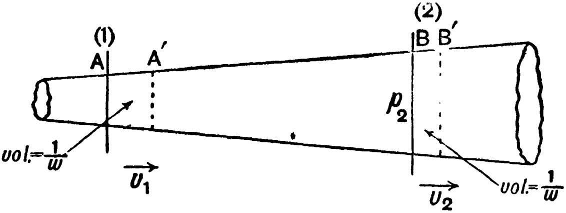

FIG. 38.—Variation of pressure and velocity.

$$\frac{p}{w}+\frac{v^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the first term representing energy transmitted by pressure and the second that carried as kinetic energy.

Now consider a horizontal pipe (Fig. 38) in which the section is not everywhere the same. While 1 lb. of water passes section (1), 1 lb. also passes section (2). The work done *on* the water A B at (1) is $\frac{p_1}{w}$ ft.-lb. per lb. and the work done *by* the water A B at (2) is $\frac{p_2}{w}$. The net work done by the water is therefore $\frac{p_2}{w}-\frac{p_1}{w}$ or $\frac{p_2-p_1}{w}$ per lb. of water passing a given point; and from the principle of the conservation of energy, this must be at the expense of its kinetic energy, which is reduced from $\frac{v_1^2}{2g}$ to $\frac{v_2^2}{2g}$ or

$$\frac{v_1^2}{2g}-\frac{v_2^2}{2g}=\frac{p_2}{w}-\frac{p_1}{w} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which may be written

$$\frac{v_1^2}{2g}+\frac{p_1}{w}=\frac{v_2^2}{2g}+\frac{p_2}{w} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

By reference to steel structures and piston rods above we have indicated that it would be misleading to say that equation (4) indicated equal energy in each pound of water. It would be better to say, as indicated by (3), that per pound of water passing between any two sections, the loss of kinetic energy is equal to the net work done *by* the water, or the gain of energy, where it is accelerating, is equal to the net work done *on* the water. Note that if there is no *flow*, no energy is transmitted—only force. Movement is an essential part of the transmission of the energy.

Bernoulli's Theorem. For a pipe which is not horizontal, we must add the net work done by or against gravity as the case may be, viz. 1 ft.-lb. per lb., per foot difference of level. Thus to lift 1 lb. of water from a level h_1 ft. above a datum at (1), to a level h_2 at (2) (a vertical distance of (h_2-h_1) ft.), requires (h_2-h_1) ft.-lb. of work per pound, which must be added to the right-hand side of (3) (as additional work done at the expense of the K. E. per pound) making the equation.

$$\frac{v_1^2}{2g}-\frac{v_2^2}{2g}=\frac{p_2}{w}-\frac{p_1}{w}+h_2-h_1 \quad . \quad . \quad . \quad (5)$$

and this makes (4) become

$$h_1+\frac{v_1^2}{2g}+\frac{p_1}{w}=h_2+\frac{v_2^2}{2g}+\frac{p_2}{w} \quad . \quad . \quad . \quad (6)$$

This is known as Bernoulli's theorem.

If the equations (4) and (6) do not indicate equal energy *contained in* 1 lb. of water passing sections (1) and (2), **what equality do they indicate?** We have seen that water moving vertically upward as a jet with velocity v_1 ft. per sec., with no friction, would reach a height of $v_1^2/2g$ ft. We may call $v_1^2/2g$, then the kinetic head at section (1). And water issuing freely from a stationary supply at p_1 lb. per sq. ft. into the atmosphere would have a velocity of $v=\sqrt{\frac{2gp_1}{w}}$, or sufficient to raise it to a vertical height $\frac{p_1}{w}$ ft. The quantity p_1/w is then called a "pressure head" and the total head in feet at section (1) is $h_1+\frac{v_1^2}{2g}+\frac{p_1}{w}$ and this also equals $h_2+\frac{v_2^2}{2g}+\frac{p_2}{w}$, so that Bernoulli's theorem expressed in symbols in equation (6) may be stated in words as follows. In a pipe full of a flowing frictionless liquid, the total head is constant. The total heads at sections (1) and (2) are made up of (a) gravitational head (above an arbitrary datum level); (b) kinetic head, and (c) pressure head. It may be well to emphasise that the terms in the expressions (5), (6), etc., are not in amounts of energy. They are not foot-pounds but foot-pounds *per pound*, which dimensionally are lengths in *feet*. Similarly $1/w$ is not dimensionally volume in cubic feet but volume in cubic feet *per pound*.

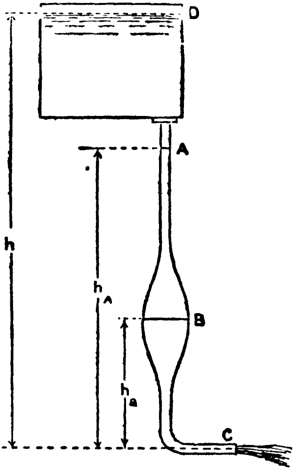

FIG. 39.—Variation of pressure with height and velocity.

The relation embodied in equation (6), which has a wider application than to flow in a pipe, is known as Bernoulli's Theorem.

In Fig. 39 we show the conditions at four places

A, B, C, and D. At A and B, the total heads consist of all three parts, while at C the head is wholly kinetic, and at D it is wholly gravitational. The total head in all cases is h and the total heads at A, B, C, and D are made up as follows:

Place	Gravitational head	Kinetic head	Pressure head	Total head
D	h	0	0	h
A	h_A	$\frac{v_A^2}{2g}$	$\frac{p_A}{w}$	$h_A+\frac{v_A^2}{2g}+\frac{p_A}{w}=h$
B	h_B	$\frac{v_B^2}{2g}$	$\frac{p_B}{w}$	$h_B+\frac{v_B^2}{2g}+\frac{p_B}{w}=h$
C	0	$\frac{v_C^2}{2g}$	0	$\frac{v_C^2}{2g}=h$

The total head is the same in each case and if p is in pounds per square foot, $w=62{\cdot}4$ lb. per cu. ft., v is in feet per second, and g is 32·2 ft. per sec. per sec.

Many interesting points arise from the foregoing simple relations but we cannot pursue them far at this stage. But we may again refer to equation (3) for two different sections of a tube conveying water in steady flow, viz.

$$\frac{v_1^2}{2g}-\frac{v_2^2}{2g}=\frac{p_2-p_1}{w}$$

If the section (1) is a narrow channel or "throat" in a tube which is elsewhere of the larger section, as at (2), v_1 can be made greatly in excess of v_2; then v_1^2 greatly exceeds v_2^2 and consequently p_2 greatly exceeds p_1. Thus a "throat" in a tube conveying fluid can be made to have a pressure below atmospheric and to give a suction. This gives just a glimpse of the principle of the injector which takes in water at atmospheric pressure and forces it into a boiler at much higher pressure, and of the choke tube of a carburettor which at the throat of a tube conveying air has a low pressure to assist the evaporation of the petrol, which as a vapour then joins the stream of air, providing a mixture for combustion in the cylinder of a petrol engine. A measured pressure difference between the throat and the full sized tube (Fig. 40) provides the means for determining the velocity of flow and hence the volume per second flowing in a pipe conveying water. Such is the principle of the Venturi water meter. (See Example 2 below.)

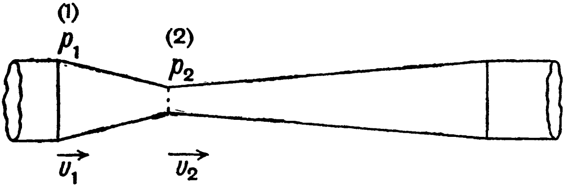

FIG. 40.—Reduction of pressure at throat.

Example 1. A horizontal pipe 0·2 sq. ft. in sectional area conveys 0·4 cu. ft. of water per sec. It has a portion converging to a minimum section of 0·01 sq. ft. followed by a gradually diverging portion leading again to the full size. If the pressure in the main pipe is 20 lb. per sq. in. above atmospheric, find the pressure at the minimum section.

Using Fig. 40 for reference

$$v_1 = \frac{0{\cdot}4 \text{ cu. ft./sec.}}{0{\cdot}2 \text{ sq. ft.}} = 2 \text{ ft./sec.}$$

$$v_2 = \frac{0{\cdot}4 \text{ cu. ft./sec.}}{0{\cdot}01 \text{sq. ft.}} = 40 \text{ ft./sec.}$$

Gain of kinetic head

$$= \frac{v_2^2 - v_1^2}{2g} = \frac{(40^2 - 2^2)(\text{ft./sec.})^2}{2 \times 32{\cdot}2 \text{ ft./sec.}^2}$$

$$= \frac{1596}{64{\cdot}4} \text{ ft.} = 24{\cdot}78 \text{ ft.}$$

This from equation (3) or (4), etc., is equal to the loss of pressure head or, since $p_1=20$ lb. per sq. in. ×144 sq. in. per sq. ft. =2880 lb. per sq. ft.,

$$\frac{p_1}{w}-\frac{p_2}{w}=\frac{(2880-p_2)}{62{\cdot}4\text{ lb./cu. ft.}}=24{\cdot}78\text{ ft.}$$

$$2880-p_2=24{\cdot}78\times62{\cdot}4\text{ lb./sq. ft.}$$

$$=1546\text{ lb./sq. ft.}$$

$$p_2=2880-1546=1334\text{ lb./sq. ft.}$$

$$\text{or }9{\cdot}26\text{ lb./sq. in.}$$

If the pressure in the pipe had been only 20 lb. per sq. in. absolute, the pressure at the throat would have been below atmospheric (14·7 lb. per sq. in.) and a pressure gauge would have shown a suction there.

Example 2. With the tube and throat dimensions of Example 1, find the flow in cubic feet per second if a mercury U-tube (or manometer) connected to the full-sized pipe and the throat shows a difference of pressure represented by a difference of level of mercury of 10 in.

Since mercury is 13·6 times as heavy as water, 10 in. of mercury represents a head of 13·6 ×10 in. =136 in. or 11·33 ft. head of water, which we need not reduce to pounds per square feet.

Since the area of section at the throat is, 0·01/0·2 or $\frac{1}{20}$, of the full section of the pipe, and since the total flow at both sections is the same, the velocity v_2 must be 20 times v_1; hence the gain in kinetic head is

$$\frac{v_2^2}{2g}-\frac{v_1^2}{2g}=\frac{(20v_1)^2-v_1^2}{2g}=\frac{399v_1^2}{2g}$$

and this is equal to the loss of pressure head of 11·33 ft

i.e. $$\frac{399v_1^2}{64{\cdot}4\text{ ft./sec.}^2}=11{\cdot}33\text{ ft.}$$

$$v_1^2=\frac{11{\cdot}33\times64{\cdot}4}{399}\text{ ft.}^2/\text{sec.}^2$$

$$=\frac{729{\cdot}5}{339}\text{ (ft./sec.)}^2=1{\cdot}828\text{ (ft./sec.)}^2$$

$$v_1=1{\cdot}352\text{ ft./sec.}$$

Volume flowing per second is of length of 1·352 ft. and of section 0·2 sq. ft.

$$1{\cdot}352 \text{ ft.} \times 0{\cdot}2 \text{ sq. ft.} = 0{\cdot}27 \text{ cu. ft.}$$

46. Force exerted by a Jet of Water. The moving water in a stream or jet has a certain definite momentum. If the water is turned from its motion in a straight line we know from the first law of motion (Art. 17) that force must be exerted upon it. And from the second law of motion (Art. 22) we know that this force is measured by the rate of change of momentum of the stream and is in the direction of the change of momentum. And the third law tells us that the force exerted on the stream by any obstacle which serves to change its momentum is equal and opposite to the force exerted by the stream or jet on the obstacle.

The force exerted by a jet of water on, say, a fixed flat plate therefore gives a peculiarly good and quite simple illustration of the three laws of motion. If we know the quantity of water flowing and its velocity, we can easily calculate its momentum. The force exerted in a direction opposite to that of flow, *by* an obstacle, will depend upon the degree of deviation of the stream of water; it may be through any angle from zero to complete reversal of direction or 180°. The change of momentum and the force are of course vector quantities, and in any but simple deviations it can best be estimated by vectors.

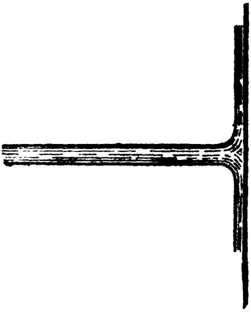

Fig. 41.—Force of a jet on a fixed plate.

Suppose a steady stream of water in the form of a horizontal jet strikes a vertical fixed flat plate, as in Fig. 41, and the water flows off at right-angles to its original direction, i.e. it is deviated through an angle of 90°. The water in the jet undergoes a complete loss of momentum in the original direction of motion, that is, an equal gain of momentum in the direction opposed to the motion and the rate of this gain measures the force exerted *upon the water by the plate* in the direction opposite to that of flow. Thus suppose the velocity of the jet to be 30 ft. per sec. and that 50 lb. of water are carried to the surface of the plate per sec., then the change (loss) of momentum in the original direction per second is

$$\text{mass} \times \text{velocity} = \frac{50 \text{ lb.-force}}{32{\cdot}2 \text{ ft./sec.}^2} \times 30 \text{ ft./sec.}$$

or rate of change of momentum

$$= \frac{50 \times 30}{32{\cdot}2} \; \frac{\text{lb.-force} \times \text{ft./sec/sec.}}{\text{ft./sec.}^2}$$

$$= \frac{1500}{32{\cdot}2} \text{ lb.-force} = 46{\cdot}6 \text{ lb.}$$

If the stream were deviated through an angle θ, the final momentum in the original direction would be $mv \cos \theta$ and the loss in the original direction would be $mv - mv \cos \theta$ or $mv(1 - \cos \theta)$, which varies from 0 to $2mv$ (when $\theta = 180°$). For deviation at right-angles, $\theta = 90°$ $\cos \theta = 0$, and the loss of momentum in the direction of flow is mv.

Finally it is to be remembered that the rate of gain of momentum measures the force exerted on the water, and the opposite reaction, equal to the rate of loss of momentum in any direction, therefore measures the force exerted by the jet.

We may note that for a frictionless liquid (or for water in so far as we can ignore the friction between it and a plate), though there is force exerted by the jet on the fixed plate, no work is done and no energy is lost. The water flows on with undiminished speed and kinetic energy despite its change in direction. In machines such as water wheels and turbines for absorbing the energy in moving water, a stream of water exerts force on a *moving* body ; it does work upon it and loses some of its energy. In the following chapters we shall see that similarly the molecules of a gas striking the fixed walls of a containing vessel exert pressure but lose no energy. Striking a moving piston which is receding, they do work upon it and lose some of their kinetic energy.

Example. A jet carries 1·5 cu. ft. of water per sec., is circular in cross-section and 2 in. diam. It impinges at right-angles on a fixed plate. Find the force exerted by the jet on the plate.

$$\text{Area of section of jet} = 4\pi/4 = 3{\cdot}142 \text{ sq. in.}$$
$$= 0{\cdot}0128 \text{ sq. ft.}$$

$$\text{Velocity} = \frac{1{\cdot}5 \text{ cu. ft./sec.}}{0{\cdot}0218 \text{ sq. ft.}} = 68{\cdot}8 \text{ ft./sec.}$$

$$\text{Weight of water flowing per sec.} = 1{\cdot}5 \text{ cu. ft.} \times 62{\cdot}4 \text{ lb./cu. ft.}$$
$$= 93{\cdot}6 \text{ lb.}$$

$$\text{Momentum per sec.} = \frac{93{\cdot}6 \text{ lb.-force}}{32{\cdot}2 \text{ ft./sec.}^2} \times 68{\cdot}8 \text{ ft./sec.}$$

$$\text{Rate of change of momentum} = \frac{93{\cdot}6 \times 68{\cdot}8}{32{\cdot}2}\ \frac{\text{lb.-force} \times \text{ft./sec./sec.}}{\text{ft./sec.}^2}$$
$$= 199{\cdot}8 \text{ lb., say } 200 \text{ lb.}$$

This is the force exerted (backwards) by the plate on the stream of water and also the magnitude of the (forward) force exerted by the water on the plate.

If the flat plate were replaced by a hemispherical bowl or a plate bent into the form of a half cylinder, which would approximately reverse the direction of flow, the change of velocity would be $v-(-v)=2v$. The rate of change of momentum would be twice as great as for the flat plate and the force exerted by and on the stream of water would be 400 lb.

Summary of Chapter IV

The mobility of the molecules in the three states of matter correspond with the essential characteristic differences which are important in the application of the principles of mechanics to the different states. Solids have immobile molecules in contact and adhering, liquids have mobile molecules sliding or rolling in contact, while in gases the molecules are in flight, with occasional collisions.

In liquids, the intensity of normal pressure at any point is equal in all directions and increases, per square foot, at a rate w per ft. of depth, where w = weight per cubic foot, or $p_2 - p_1 = wh$.

The total pressure on a submerged surface is equal to its area multiplied by the intensity of pressure at its centre of gravity.

Liquids transmit force and energy like solids except that they cannot transmit appreciable tension in any case, nor shear force at low speeds. Water conveniently transmits pressure and energy for lifting and other purposes. The work done for a volume V cu. ft. passing at constant pressure p lb. per sq. ft. is pV ft.-lb.

Suction is caused by the difference between the atmospheric and a lower pressure. The suction pump is thus limited in its suction lift to less than 34 ft. head of water, corresponding to the pressure of the atmosphere. The force pump is not limited in any such way.

The flow from an orifice is $ka\sqrt{2gh}$ cu. ft. per sec., where k is the coefficient of discharge and h is the head of *water*. Under any pressure p, the velocity of issue is $\sqrt{2gp/w}$.

In a frictionless liquid flowing in a pipe, the total head,

$$h+\frac{v^2}{2g}+\frac{p}{w}$$

is constant. Or the gain of kinetic energy per pound of water flowing between two sections is equal to the work done on the water, or

$$\frac{v_2^2-v_1^2}{2g}=\frac{p_1-p_2}{w}+h_1-h_2$$

that is

$$\frac{v_2^2}{2g}+\frac{p_2}{w}+h_2=\frac{v_1^2}{2g}+\frac{p_1}{w}+h_1$$

The force exerted by a jet of water on a fixed body is equal to the rate of change of momentum of the water and in the opposite direction to the gain of momentum; in any direction, it is equal to the rate of *loss* of momentum by the water in that direction.

EXAMPLES IV

1. What water pressure, in pounds per square inch, will support a head of mercury of 40 in. above the pressure of the atmosphere? Specific gravity of mercury 13·6.

2. A dock gate has a depth of 30 ft. of sea-water on one side and is dry on the other. If it is 40 ft. wide, find the total pressure upon it, taking the weight of sea-water as 64 lb. per cu. ft.

3. Find the total pressure on a circular plate 4 ft. diameter when its centre is 7 ft. below the surface of fresh water.

4. The bottom of a rectangular tank measures 6 ft. by 4 ft. Find the total pressure on the bottom and sides of the tank when 1000 gal. of water are in it (1 gal. weighs 10 lb.).

5. In a hydraulic press the ram is 10 in. in diameter, and the plunger of 1 in. diameter is worked by an effort of 30 lb. on a handle which gives it a leverage of 14 to 1. Assuming an overall mechanical efficiency of 80 per cent., what total pressure will be exerted by the ram?

6. How many gallons per horse-power per hour will be required from a hydraulic power main at 800 lb. per sq. in. if the efficiency is 70 per cent.?

7. A single-acting force pump has to deliver water against a head of 250 ft. Neglecting frictional losses what must be the average force exerted during the delivery stroke on the plunger, which is 4 in. diameter? If the stroke is 4 in. and there are 120 delivery strokes per minute what horse-power will be required to drive the pump if its efficiency is 85 per cent.? How many gallons will be delivered per minute if the slip is 5 per cent.?

8. A pump is driven by an electric motor of 85 per cent. efficiency and delivers 100 gal. of water per minute under a head of 125 ft. The efficiency of the pump is 80 per cent. and the motor works at 200 volts. What current will it take? If the cost of electrical energy is 1*d*. per kWh. what is the cost of running the pump per hour?

9. Find the discharge per hour from a circular sharp-edged orifice 1 in. diameter under a constant head of 12 ft. (coefficient of discharge is 0·62).

10. Find the diameter of a sharp-edged circular orifice to give a discharge of 70 gal. of water per hour under a head of 3 ft.

11. Water flows in a pipe of varying section into the atmosphere from a reservoir, the surface of which is 80 ft. above the outlet of the pipe. Neglecting friction find the velocity of the water leaving the open end of the pipe and the pressure 50 ft. above the open end if the velocity of the water at this level is 20 ft. per sec.

12. A pipe full of running water discharges 8 cu. ft. per sec. At a certain point A, the cross-sectional area is 4 sq. ft., and at B, 10 ft. below A, the cross-section is 2 sq. ft. Find the difference of pressure between A and B.

13. A horizontal jet of water 4 in. diameter moving with a velocity of 50 ft. per sec. strikes a vertical fixed plate. Calculate the force exerted by the jet on the plate. What would it be if the jet were issuing under the pressure of a head of 80 ft. of water?

CHAPTER V

MECHANICS OF GASES

47. Function of Heat Engines. Heat engines are machines for the conversion of heat energy into mechanical energy by the expansion of an elastic fluid doing work. There is much in common in the principles of heat engines, air or gas compressors and refrigerating machines; they use as their elastic fluid working medium, a gas such as air or a condensable vapour such as steam or ammonia.

Although the steam engine for a long time was the only important heat engine, we shall approach the principles of heat engines by considering the compression and expansion, first of gases such as air, which are not condensable at the working temperatures of the engines, but only at very low temperatures. This preliminary study will be clarified and made more interesting if we have a little understanding of the nature of gases, which a knowledge of mechanics makes possible.

48. Nature of a Gas. The gaseous state of matter differs greatly from the solid in which the molecules are practically fixed in position, and from the liquid in which the molecules, though mobile and moving continuously one over the other, are in continual contact with one another. In a gas the molecules are moving about at considerable speed in the space occupied by the gas, colliding with other molecules and with the walls of the vessel which confines the gas. Also the gas occupies the whole of any vessel to which it is admitted, the moving molecules being limited in their flight only by the walls.

49. Ideal Gas. The following articles (50, 51, 52, 53, 54, 55) refer to an *ideal* (sometimes called a "perfect") gas. Actual gases behave nearly as this ideal gas at temperatures much above those at which they may be liquified. The deviation from the ideal of actual gases under conditions of temperature and pressure, approaching those of liquefaction are important in more advanced branches of study and in applications. But for such gases as the main constituents of the air at ordinary atmospheric temperatures, and above, the ideal gas provides a simple and most useful basis on which to study and understand the working of many types of heat engines. The molecules of an ideal gas must be regarded as small even in comparison with the average distance through which they move between collisions one with another.

50. Pressure exerted by a Gas. A molecule of a gas although exceedingly small has definite mass and because it has also motion, it has momentum. If it strikes the wall of a containing vessel normally, that is, at right angles to the surface, its velocity will be exactly reversed in direction. If the impact is not in a normal but in an oblique direction, the speed will still be unchanged, but the normal component of the velocity will be reversed in direction. Thus, there will be, due to the reaction of the walls a change in momentum inward of the gas, and the rate of change of momentum at any given area of the boundary surface, say one square inch, will correspond to, and be equal to, the force exerted on the gas by the wall (see Arts. 22 and 36). And the gas, of course, exerts a corresponding equal outward force on the wall. The rapid succession of impacts of many millions[1] of molecules per second on every square inch may be looked upon as blending into a steady continuous force much as in the case of a jet of water turned from its course by a solid body such as a plate or a hollow cup, which serves not merely to turn the stream at right angles, but to reverse its direction.

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[1] The following average figures deduced from experiment for air at atmospheric pressure and 0° C. will illustrate the order of magnitudes: velocity of flight of molecule 500 m. per sec., diameter of molecules 5×10^{-18} cm., distance travelled between successive collisions 200 diameters, distance apart of molecules 6 diameters, number of collisions of each molecule about 5000 million per sec., number of molecules per cubic centimetre 27×10^{18}. It is almost impossible to grasp the magnitude of such numbers.

Let us consider how the pressure exerted by the gas is related to the velocity of the molecules. The velocity of different molecules will be in diverse directions, and we are concerned with some sort of mean of the components of velocity normal or perpendicular to, say, a particular square inch of wall of the containing vessel. (Actually it is the square root of the average of the square of the normal velocity of all the molecules, but this is appropriate to later study.) Let this normal velocity be v ft. per sec. and let the mass of a molecule be m, Then since the direction of flight is reversed at impact, the change of momentum during impact is $2mv$. And with a fixed number of molecules in the gas in a vessel of constant volume, the number of impacts per second on a square inch of surface will evidently also be proportional to v which may perhaps best be realised by thinking of a hollow box, cubic in shape, each internal wall being an inch square, and of the successive impacts of a particular molecule on one face, and then another, of the box. Hence, the total rate of change of momentum at the wall on any area, such as a square inch, will be proportional to v^2. For the change of momentum at each impact is proportional to v, and the number of impacts per second is also proportional to v, and since the rate of change of momentum is the measure of the force exerted, the pressure of the gas per square inch is proportional to v^2.

51. Heat Energy of a Gas. The heat energy of a gas consists of the kinetic energy of its molecules. If the weight of a molecule is w and its velocity is v, its kinetic energy due to its flight is $\frac{1}{2}\frac{w}{g}v^2$, and if we regard the total kinetic energy of the molecule as being proportional to it, thiskinetic energy of flight will be the only one we need contemplate (i.e. we can neglect any kinetic energy of spin or vibration) in an ideal gas. Then for a fixed mass of gas, having therefore a constant number of molecules, each of constant weight w, the kinetic energy of the gas will be proportional to v^2. But the heat energy of a gas, like that of a solid or liquid, must be changed in proportion to changes in temperatures provided it has a constant specific heat. Hence changes in v^2 are proportional to changes in temperature.[1]

52. Boyle's Law. We are now in a position to understand this well known law, not only as an approximate relationship established by simple experiments, but also the reasons for that relationship as made clear by the nature of the gas, and the consequences shown in the preceding articles. The law derives its name from the seventeenth-century Irish scientist Robert Boyle. In common usage, the word *pressure* of a fluid is taken to mean the intensity of pressing or thrusting force or force *per unit of area*, and not the whole or total force on some particular area. Boyle's Law may be stated in words as follows:

The volume of any given mass of a gas varies inversely as its pressure if the temperature remains constant.

Or in symbols, if p is the pressure (intensity) and V is the volume of any mass of gas,

$$\mathrm{V} \propto \frac{1}{p} \text{ or } p \times \mathrm{V} = \text{constant} \quad . \quad . \quad . \quad (1)$$

or

$$p_1\mathrm{V}_1 = p_2\mathrm{V}_2 \quad . \quad . \quad . \quad . \quad (2)$$

where the suffixes 1 and 2 denote the initial and final conditions respectively.

Accepting the principles applicable to an ideal gas set forth in Arts. 50 and 51, it is clear why this relationship between p and V must hold. For suppose (Fig. 42) a volume of gas V in a cylinder to be pressed by a gas-tight piston into a space $\frac{1}{2}$V. If the temperature remains constant, it follows from Art. 51 that the velocity v of the molecules must remain constant (for the kinetic energy $\frac{1}{2}mv^2$ is constant, and therefore v^2 remains constant). But as the molecules are twice as densely crowded in half the space they formerly occupied, the number of impacts per second on any square inch will be twice as many as the number before the gas was compressed. That is, the force exerted per square inch will be doubled. Similarly if the gas be compressed into, say, one third of its former volume the pressure will be trebled, and so on. Thus the flight of the molecules at constant (average) velocity, corresponding to constant temperature, gives a simple explanation of the validity of Boyle's Law for an ideal gas. Changes made at constant temperature are called *isothermal* changes.

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[1] Experimental facts confirm that in actual gases the heat energy is proportional to the molecular energy of flight.

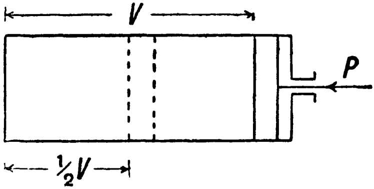

FIG. 42.—Isothermal compression.

Example. If 4 cu. ft. of air at 15 lb. per sq. in. be compressed to 0·5 cu. ft. at constant temperature, find the pressure. Also find the pressure when the volume is 3·5, 3·0, 2·5, 2·0, 1·5 and 1·0 cu. ft. and plot a graph showing the pressure for any volume between 4·0 and 0·5 cu. ft.

From the statement (2) in symbols, if p_2 be the pressure in pounds per square inch at $V = 0{\cdot}5$ cu. ft.

$$p_2 \times 0{\cdot}5 \text{ cu. ft.} = 15 \text{ lb. per sq. in.} \times 4 \text{ cu. ft.}$$

$$p_2 = \frac{15 \text{ lb. per sq. in.} \times 4 \text{ cu. ft.}}{0{\cdot}5 \text{ cu. ft.}}$$

$$= 120 \text{ lb. per sq. in.}$$

In the remaining calculations we may safely omit the units and write

$$pV = 15 \times 4 = 60$$

$$p = \frac{60}{V}$$

and if V be replaced by its numerical values, p will be in pounds per square inch. The result is as follows :

V cu. ft. . .	4·0	3·5	3·0	2·5	2·0	1·5	1·0	0·5
p lb. per sq. in.	15	17·1	20	24	30	40	60	120

The relation of p to V is shown in Fig. 43. The graph is a rectangular hyperbola and expansion according to the law pV=constant, is sometimes called "hyperbolic expansion."

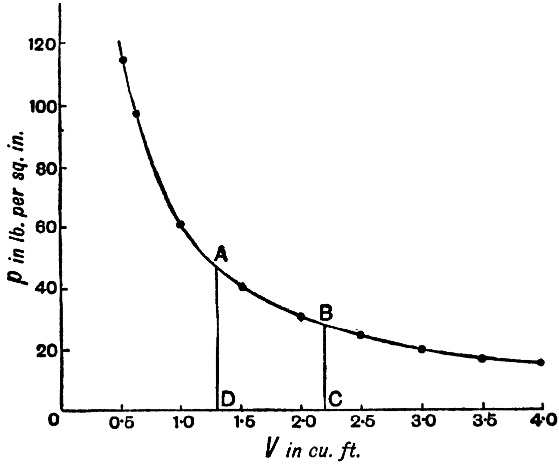

FIG. 43.—Hyperbolic expansion.

The product pV. The product pV, which is constant for isothermal expansion or compression of an ideal gas is of importance. If we take the volume V as that of a cylinder A sq. in. in section and l in. long, expressed in cubic inches, so that $V = l \times A$ cu. in., then for a pressure p lb. per sq. in., $p \times A$ lb. would be the force exerted by a gas on a piston of A sq. in. area just fitting the cylinder bore, and if this piston made a stroke of l in. under constant pressure p the work done would be pA lb. $\times\, l$ in. $= pAl$ in.-lb. $= pV$ in.-lb. Thus the product pV would give the work done in *inch-pounds* if behind the piston a volume V cu. in. of gas were evolved from no volume (or admitted from a reservoir of gas) at a constant pressure p lb. per sq. in. Note that correspondingly if p were in pounds per square *foot* and V were in cubic *feet*, $p\text{V}$ would represent the corresponding work done in *foot-pounds* as in (6), Art. 41.

Work Diagram. The area included between the graph in Fig. 43 and the base line and bounded by any pair of vertical ordinates, such as AD and BC, would be the diagram of work done on the gas in the corresponding stage of the compression, or of work done by the gas during corresponding expansion. To find the scale on which the area represented work, we should know how many pounds per square foot were represented by 1 in. vertically and how many cubic feet were represented by 1 in. horizontally and then the product of these two numbers would give the number of foot-pounds represented by 1 sq. in. of area on this diagram.

53. Charles's Law. This law (named after the French physicist Jacques Alexander Cesar Charles, 1746–1823, who was the first to employ hydrogen for the inflation of balloons—1783) states, as a result of experiment, that if the pressure remains constant, for equal increases in temperature a given mass of gas expands by equal amounts in volume.

Note that the increases are not by equal proportions of its volume but by equal volumes. A similar rule gives contractions in volume for decreases in temperature.

The increase (or decrease) is $\frac{1}{273}$ part of its volume at 0° C. per 1° C. change in temperature or $\frac{1}{492}$ part of its volume at 32° F. per 1° F. change in temperature.

If a gas continued to behave in this way as its temperature fell from 0° C. at constant pressure, it would have been reduced, say, in 272° (at a temperature of −272° C.) by $\frac{272}{273}$ of its volume, and its volume at −273° C. would be zero.

Or putting the matter in symbols, the volume at any temperature t° C. will be $\text{V}=\text{V}_0\left(1+\frac{t}{273}\right)$ or $\text{V}_0\frac{t+273}{273}$, where V_0 is the volume at 0° C. This gives a volume zero at $t=-273°$ C. Actual gases will not follow the law exactly at low temperatures when the molecules will be much closer to one another, but at higher temperatures the rule is approximately correct.

Correspondingly on the Fahrenheit scale, the expansion of a gas at constant pressure is $\frac{1}{492}$ part of its volume at 32° F. for each 1° F. increase of temperature and at any temperature t°F., $V=V_0\frac{t+460}{492}$.

The temperature, $-273°$ C. or $32-492=-460°$ F., which would make the volume of any gas zero if it continued to follow Charles's Law at very low temperatures (which actual gases do not), is called the absolute zero of temperature, and temperatures reckoned from this level such as $t+273°$ C. or $t+460°$ F. are called *absolute* temperatures and are usually represented by the symbol T. Thus, in symbols, Charles's Law for gases at constant pressure is

$$V \propto T \quad \text{or} \quad V=T\times\text{constant}.$$

Work done in expansion at constant pressure.

Just as pV represents the work done from zero volume at a constant pressure p, so the work done in expansion at constant pressure p from a volume, say, V_1 to a volume V_2 will be $p(V_2-V_1)$, and if p is in pounds per square *foot* and V_2 and V_1 in cubic feet, the work done would be in foot-pounds.

✓ **54. Gas Heated at Constant Volume.** If we consider a fixed weight of gas in a cylinder or other vessel entirely closed so that the volume of the gas cannot be changed (as by the movement of a piston), then the effect of heating the gas, say, by conduction from an outside source of heat, will be to increase its pressure, and the effect of cooling it will be to decrease its pressure. The way in which the pressure changes with change in temperature at constant volume is easily found as a matter of algebra from the relations established for the Laws of Boyle and Charles, for if $V \propto T$ (Charles's Law) when p

is constant and $V \propto \frac{1}{p}$, when T is constant it follows[1] that (as shown in books on elementary algebra)

$$pV \propto T \text{ or } pV = T \times \text{constant} \quad . \quad . \quad . \quad (1)$$

or that, if p_1 is the pressure and V_1 is the volume of a quantity of gas at an absolute temperature T_1, then at any other pressure p_2 and absolute temperature T_2, the volume V_2 will be

$$V_2 = \frac{p_1}{p_2}\frac{T_2}{T_1}V_1 \text{ or } \frac{P_2V_2}{T_2} = \frac{p_1V_1}{T_1} \quad . \quad . \quad . \quad (2)$$

which may also be written

$$\frac{pV}{T} = R \text{ (a constant)} \quad . \quad . \quad . \quad . \quad (3)$$

or if V remains constant, $\frac{p_2}{p_1} = \frac{T_2}{T_1}$. That is, at constant volume, the pressure of a gas is proportional to its absolute temperature (just as the volume is at constant pressure). Moreover the change in pressure at constant volume is at the rate of $\frac{1}{273}$ per degree C., or $\frac{1}{492}$ per degree F., of the pressure at the temperature of freezing water (0° C., 32° F.) or $p = p_0\frac{T}{273}$ on the Centigrade scale or $p = p_0\frac{T}{492}$ on the Fahrenheit scale.

That this rate of change of pressure with temperature at constant volume actually holds good has been proved by independent experiment as well as by deduction algebraically from Charles's Law.

It is very easy to see why in an ideal gas the pressure rises in proportion to the rise of temperature if the volume is constant, for we have seen in Art. 50 that both the heat energy (kinetic) and the pressure contributed by a single molecule are proportioned to v^2. So that if the volume remains constant (and consequently the number of molecules per cubic inch and therefore the mass per cubic inch), the pressure is proportional to v^2. And since v^2 is proportional to the absolute temperature, the pressure is proportional to the temperature.

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[1] For while p remains constant at p_1 let T_1 be changed to T_2 and the volume V_1 consequently change to $V_1' = V_1 \times \frac{T_2}{T_1}$. Then while T remains constant at T_2 let the pressure be changed to p_2 and the volume then becomes, $V_2 = V_1' \times \frac{p_1}{p_2} = V_1 \times \frac{T_2}{T_1} \times \frac{p_1}{p_2}$, hence $\frac{p_2V_2}{T_2} = \frac{p_1V_1}{T_1}$, or $pV/T = \text{constant}$.

If a gas is heated it is clear that the pressure will just remain constant if the volume expands in the proportion to T, for then the reduction of molecules per cubic inch will just counteract the greater and more frequent impacts made by each molecule, which contributes to the pressure an amount proportional to v^2, that is to T.

55. Energy required to Heat or Expand a Gas. (a) *At Constant Volume.* If a gas be heated in a closed vessel so that it cannot expand, the temperature rises by an amount proportional to the increase of kinetic energy of every molecule and therefore to the mean value of v^2, where v is the velocity of any molecule. And if we know the specific heat of the gas, we can easily find the heat required to raise the temperature of any given weight of gas any number of degrees, viz. heat required = specific heat × mass of gas × rise of temperature.

If C^v = specific heat of gas at constant volume

t_1 = initial temperature

t_2 = final temperature

w = mass of gas in pounds,

$$\text{heat required} = C_v \times w(t_2 - t_1) \quad . \quad . \quad . \quad (1)$$

B.Th.U. if t is in degrees F. or lb.-C.H.U. if t is in degrees C.

This heat energy is sometimes called an addition to the internal energy of the gas. It is an increase in the kinetic energy $\left(\frac{1}{2}\frac{w}{g}v^2\right)$ of the molecules which move faster, the mean value of v^2 being proportional to T, the absolute temperature.

(b) *Heat required at Constant Pressure.* If instead of being confined in a vessel of constant volume, the gas can expand and drive outwards the walls of the vessel that contains it, the heat required to raise the temperature of the gas will be greater than that required at constant volume. For the gas does work against external pressure and thus parts with energy. It must therefore receive more energy in order to raise its temperature from $t_1°$ to $t_2°$ than it would require at constant volume, when no work is done against external pressure. This work done against external pressure is perhaps more easily understood if we consider gas in a cylinder (Fig. 44) so that the whole of the expansion at constant pressure takes place in one direction by the gas driving upwards a frictionless piston to which a total downward force P is applied (including the pressure of the atmosphere, the weight of the piston itself, and any other added load). Then if A is the area of the piston in square *feet* and if the piston moves l ft. while the gas is heated from, say, $t_1°$ to $t_2°$ and expands from a volume V_1 cu. ft. to V_2 cu. ft.,

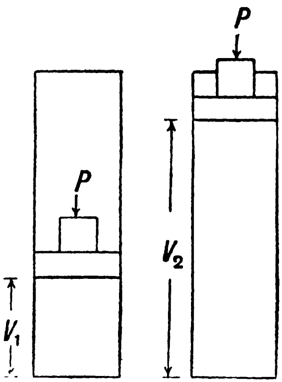

FIG. 44.—Gas heated at constant pressure.

$$\begin{aligned} \text{work done} &= P \times l \text{ ft.-lb.} \\ &= pAl \text{ ft.-lb.} \\ &= p(V_2 - V_1) \text{ ft.-lb.} \quad . \quad . \quad . \quad (2) \end{aligned}$$

where $p = P/A$ = pressure in lb. per *sq. ft.*, under which the heating takes place.

This work done against external pressure amounts to a considerable portion of the energy required to heat a gas at constant pressure. In air, for example, it amounts to 0·4/1·4 or $\frac{2}{7}$ of the whole, or in other words the total heat required is about 1·4 times as much as that required at constant volume or 1·4 times as much as that portion which, at constant pressure, goes to increase the internal energy of the gas (i.e. the kinetic energy of its constituent molecules).

To put the matter another way, the effect of allowing a gas to expand is to cool it; and unless more heat energy is supplied than is necessary to increase the internal (kinetic) energy, the temperature will not rise as much as it would in the case of a gas enclosed in a vessel of constant volume.

Consider why expansion of a gas has a cooling effect. One can say that as it does work in expansion it loses energy and therefore must be cooled, but we can, from our knowledge of mechanics, understand the action more fully. In a closed vessel of constant volume, a molecule of the contained gas, striking the wall with a normal velocity v, rebounds also with a normal velocity v in the reverse direction. But if the vessel is not of constant volume and the envelope is expanding, the walls are not stationary but receding from the impinging molecules, e.g. in Fig. 44 the piston only was receding from the flying molecules of the gas in the cylinder. The molecules striking a receding wall have, with respect to it, a relative velocity less than their velocity relative to a stationary wall, and they rebound from it (see Art. 36) with a reversed normal velocity, less than they would acquire by collision with a stationary wall.[1] By impact with a receding body they lose speed and kinetic energy. That is, unless heat is supplied, a gas will be cooled by expansion. Hence to attain a given increased temperature t_2, more energy must be supplied than is necessary if the gas is heated in a vessel of constant volume. The rate at which heat is supplied, and at which consequent expansion takes place is unimportant. If the piston moves quickly the loss of velocity of a molecule per impact is greater but the number of impacts made during the expansion is accordingly smaller. In terms of energy, without reference to change in molecular velocities, if the gas does work on the receding body it will require more energy to raise its temperature than if, at constant volume, it does no work.

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[1] If v' is the velocity of the receding piston, the (normal) velocity of the molecule relative to the piston is $v - v'$, and if it rebounds with a normal velocity $-(v - v')$ relative to the piston its absolute normal velocity will be $-(v - 2v')$ or $v - 2v'$ in a direction opposite to the original direction of v.

If, instead of heating from a temperature t_1 to a higher temperature t_2 at constant pressure, we consider cooling from t_2 to t_1 and reduction in volume from V_2 to V_1, work is done *upon* instead of *by* the gas and to a corresponding extent more heat energy is given out than in the case of cooling at constant volume. In this case a molecule impinging on an approaching piston rebounds with its speed and kinetic energy increased, but we shall return to this point when considering compression (Art. 57).

Making a statement in symbols for the case of heating the weight w of gas, if C_p = specific heat at constant pressure

$$\text{heat required} = C_p \times w(t_2 - t_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in B.Th.U. if t is in degrees F., or in lb.-C.H.U. if t is in degrees C.

But in the case of the ideal gas (where we need consider only the kinetic energy of flight and can neglect any work done due to forces of attraction between molecules at relatively large distances apart), the gain in *internal* energy is the same whether the heating takes place at constant volume or at constant pressure and the heat energy supplied in the latter exceeds that for constant volume by the heat equivalent to the amount of work done, that is by $\frac{1}{J} \times p(V_2 - V_1)$, where J is Joule's equivalent in foot-pounds of one thermal unit, i.e. of one B.Th.U. or one lb.-C.H.U. according as t_1 and t_2 are measured on the Fahrenheit or Centigrade scales of temperature.

Thus equation (3) becomes

$$\text{Heat required} = C_p w(t_2 - t_1)$$
$$= C_v w(t_2 - t_1) + \frac{1}{J} p(V_2 - V_1) \quad . \quad (4)$$

Or in words

$$\text{Heat supplied} = \text{Increase of internal energy} + \text{external work done} \quad . \quad (5)$$

the same appropriate units being used for each quantity. This equation, which is merely a statement of the conservation of energy, is of fundamental importance. We have considered it in relation to expansion at constant pressure, but that is only a special case and the equation is of much more general application. For if the volume changes in *any* way during the supply of heat energy the pressure changes accordingly, and if we can draw the graph of p for all values of the volume V we can, as in Art. 52, calculate the external work done in expansion and thus, from (5), the heat required to affect such a change in pressure, temperature, and volume. Thus the specific heat of a gas depends upon how p and V are varying during the change of temperature. C_v is the most important value of the specific heat; it corresponds to a change only in the internal energy. C_p is an important value for various physical reasons, for a gas may often be heated under constant pressure, or a vapour may be formed from a liquid under constant pressure. But since a gas can be heated under many conditions such as an increase of pressure together with some increase in volume, there are an indefinite number of possible specific heats for any one gas, but C_v and C_p are of special importance.

Before leaving equation (5), of which equation (4) is the special case for constant pressure, let us look at two other special cases.

✓ (c) *Heat Supplied in Isothermal Expansion.* In this case the temperature t_2 remaining the same as t_1, the increase of internal energy is nil, so that equation (5) becomes

$$\text{Heat supplied} = \text{work done} \quad . \quad . \quad . \quad (6)$$

the work done being calculated as in Art. 52 for (hyperbolic) expansion according to Boyle's Law. In isothermal compression the work done is equal to the heat which must be taken away from the gas to keep its temperature constant, that is, to prevent a *rise* of temperature.

(d) *Expansion with no Supply of Heat.* If no heat is supplied to a gas expanding and doing work the expansion is said to be *adiabatic*, a condition which would be ideally realised by the use of a cylinder and piston of absolutely non-conducting materials. In this case equation (5) becomes

$$\text{Increase of internal energy} + \text{work done} = 0$$

or

$$\textit{Decrease} \text{ of internal energy} = \text{work done} \quad . \quad (7)$$

Thus, if we know the work done by the gas in adiabatic expansion we can calculate how much its temperature falls. Thus, equation (7) can be written in B.Th.U.

$$w \times C_v \times (t_1 - t_2) = \frac{1}{J} \times \text{work done in ft.-lb.}$$

or

$$t_1 - t_2 = \frac{\text{work done in ft.-lb.}}{J \times w \times C_v} \quad . \quad . \quad . \quad (8)$$

We have already seen *why* a gas will be cooled by expansion but we may repeat the reason in this important case. No heat being supplied, the molecules colliding with a receding wall of the containing vessel will rebound with diminished speed, that is, their kinetic energy will be diminished and the temperature of the gas will be reduced. Or to put the matter from the point of view of energy only, if the molecules impinge on a fixed wall they exert force only and no work is done. But if they impinge on a receding wall or piston, they exert force on it through a distance and do work, giving energy to the moving wall or piston. Thus they rebound with diminished kinetic energy and the gas is cooled.

Example 1. 1 lb. of air at 32° F. and 14·7 lb. per sq. in. pressure occupies 12·39 cu. ft. If it is heated at constant pressure to 132° F., find its volume and the work done in foot-pounds in the expansion. State the heat equivalent to this work in B.Th.U.

$$\text{Volume at } 132^\circ \text{ F.} = 12{\cdot}39 \times \frac{460+132}{492} \text{ cu. ft.}$$

$$= 12{\cdot}39 \times \frac{592}{492} = 14{\cdot}91 \text{ cu. ft.}$$

$$\text{Increase in volume} = 14{\cdot}91 - 12{\cdot}39 = 2{\cdot}52 \text{ cu. ft.}$$

$$\text{Work done} = 14{\cdot}7 \times 144 \times 2{\cdot}52 = 5334 \text{ ft.-lb.}$$

$$\text{Equivalent heat} = 5334/778 = 6{\cdot}857 \text{ B.Th.U.}$$

Example 2. Find the total heat required to effect the expansion in Example 1 if the specific heat of air at constant volume is 0·1691 and hence find the specific heat at constant pressure and the ratio to that at constant volume.

$$\text{Increase in temperature} = (132^\circ - 32^\circ)\text{ F} = 100^\circ \text{ F.}$$

Heat required per pound at constant volume $= 100^\circ \text{ F.} \times 1 \text{ lb.} \times 0{\cdot}1691$
$= 16{\cdot}91$ B.Th.U.

Additional heat per pound for external work at constant pressure (from Ex. 1) $= 6{\cdot}86$ B.Th.U.

Total heat required per pound at constant pressure $= 23{\cdot}77$ B.Th.U.

Specific heat at constant pressure $= \dfrac{23{\cdot}77 \text{ B.Th.U.}}{1 \text{ lb.} \times 100^\circ \text{ F.}} = 0{\cdot}238$

$$\text{Ratio } C_p/C_v = \frac{0{\cdot}238}{0{\cdot}169} = 1{\cdot}41$$

56. Actual Gases and Vapours. All gases can be liquefied by sufficiently high pressure and low temperature; but the so-called permanent gases, such as the nitrogen and oxygen of the atmosphere, require very low temperatures and at ordinary atmospheric temperatures they cannot be liquefied by pressure alone however great. Atmospheric temperatures are far above the critical temperature for which liquefaction of such gases becomes possible by increase in their pressures.

In these circumstances the substances are called gases, while the term vapour is used for a substance in a similar state but not greatly above the temperature at which it has been evaporated from a liquid or could be condensed into a liquid. It will be noticed that the distinction between a gas and a vapour lacks precision; there is not a definite borderline of temperature for any particular substance.

Actual *gases* behave approximately in accordance with the ideal gas; they follow closely Boyle's Law and the relation pV/T=constant. When they approach the critical temperature at which they may be liquefied and may be regarded as vapours, the deviations from the laws of ideal gas becomes greater and the differences are of different kinds and amounts for different substances. Though important, the deviations of behaviour of actual gases from that for ideal gas do not concern us at this stage in the study of heat engines.

Summary of Chapter V

The molecules of an ideal gas, confined within any vessel, are flying about the space occupied by the gas, colliding with one another and with the boundary walls. The pressure exerted by the gas is that of an enormous number of such molecular impacts. It is proportional to v^2, the mean square of the velocities of the molecules, and so too is the absolute temperature of the gas, for its heat energy consists of the kinetic energy of the molecules. In the ideal gas this kinetic energy is largely that of flight and wholly proportional to it.

Boyle's Law. At constant temperature the mean velocity of flight is constant and the magnitude of the average impact is constant. The pressure is then only dependent upon the density or number of molecules per cubic inch. For any given mass of gas, the pressure is therefore inversely proportional to the volume occupied.

Charles's Law. When a gas is heated, the velocity of its molecules and the *magnitude* of their blows increase; hence if the pressure is to remain constant, its volume must increase in proportion to the absolute temperature, thereby diminishing the *number* of blows per square inch.

General Law. At constant volume, both p and T are proportional only to v^2 and therefore p is proportional to T, which also follows from the previous two laws. Hence $pV/T = \text{constant}$ or $p_1V_1/T_1 = p_2V_2/T_2$.

Expansion and Heat Energy. In expansion at a constant pressure, the work done is the product of the pressure and the increase of volume. In expansion at varying pressure, the work done can be found from a p-V graph. In any expansion the gain of energy by the molecules of a gas is the product of the rise of temperature, the mass and the specific heat at constant volume. When heating takes place with increasing volume, the heat required exceeds the gain of energy of the gas by the heat equivalent to the work done by the gas whether against constant or variable pressure.

Isothermal Expansion. If expansion takes place at constant temperature, the mean speed of the molecules remains constant and heat exactly equivalent to the work done by the gas has to be supplied. If the supply is less, the velocity of the molecules falls due to their impacts with a receding wall or boundary and the gas is cooled.

Adiabatic Expansion. If no heat is supplied during expansion the work done is wholly at the expense of the energy of the gas. The speed of the molecules rebounding from the expanding boundary is reduced and the temperature ($\propto v^2$) falls.

Actual Gases. The mechanics of the ideal gas provides a very servicable explanation of the behaviour of actual gases under conditions of temperature and pressure far removed from those of liquefaction. And even to the behaviour of a vapour, that of an ideal gas is in some respects a guide.

EXAMPLES V

1. Describe the spacing and motion of the molecules of a gas and contrast them with those in a liquid. Explain why there is no substantial difference between the internal pressure on the top and bottom of a closed vessel containing gas. Is this true in the case of a liquid ?

2. Why is the pressure of a gas increased (*a*) by reducing its volume, the temperature remaining constant, and (*b*) by raising its temperature the volume being constant ?

3. Air occupying 4 cu. ft. is compressed until its volume is 1 cu. ft. If the temperature remains constant and the initial pressure was 15 lb. per sq. in., find the final pressure and by plotting the p-V diagram to scale, find, in foot-pounds, the work done in compressing it.

4. If 2 cu. ft. of an ideal gas at 15 lb. per sq. in. and 60° F. is compressed till its volume is 0·2 cu. ft. and its temperature is then 800°F., find its pressure.

5. If an ideal gas at 15 lb. per sq. in. pressure is compressed to one-quarter of its volume and its pressure rises to 100 lb. per sq. in., find its temperature if that before compression was 100° F.

6. Air at 400 lb. per sq. in. pressure and 120° F. is heated till its pressure is 500 lb. per sq. in. and its temperature is 400° F. Find the percentage increase in volume.

7. How many cubic feet of air at 60° F. and 14·7 lb. per sq. in. pressure must be pumped into a receiver of 10 cu. ft. capacity to fill it at 200 lb. per sq. in. pressure and a temperature of 110° F. ?

8. Find the weight of 5 cu. ft. of air at 4 atmospheres pressure and a temperature of 140° F. if the weight per cubic foot of air at atmospheric pressure and 32° F. is 0·0807 lb. If this air is adiabatically expanded till its temperature has fallen to 40° F., find how many foot-pounds of work have been done if the specific heat of air at constant volume is 0·1691.

9. A volume of 10 cu. ft. of air at 14·7 lb. per sq. in. and 60° F. is heated at constant pressure to a temperature of 180° F. Find the work done in foot-pounds. If the specific heat at constant pressure is 0·2375, find the total heat required and hence find the specific heat of air at constant volume, taking the weight of air as 0·0807 lb. per cu. ft. at 14·7 lb. per sq. in. and 32° F.

CHAPTER VI

PRINCIPLE OF A HEAT ENGINE

57. Adiabatic Expansion and Compression. We have seen in Art. 55 that if a volume of ideal gas in a cylinder is expanded against a resistance, doing work, and has no heat energy supplied to it, it will be cooled. Its pressure will therefore fall more than it would in isothermal expansion, i.e. than if heat were supplied to keep up its temperature to the initial level. In Fig. 45 the way in which pressure falls off in adiabatic expansion is illustrated. If air is expanded adiabatically to twice its original volume, the pressure instead of falling to half its initial value, as it would in isothermal expansion (Boyle's Law), will fall to less than 38 per cent. of its initial value. And evidently the work done, represented by the area

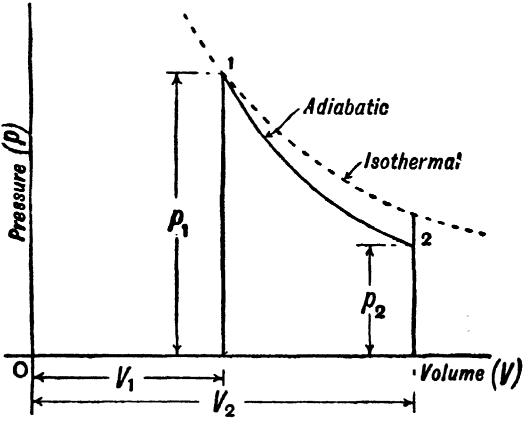

FIG. 45.—p-V diagram for adiabatic expansion.

under the adiabatic curve, is less than the work done in isothermal expansion. But the main fact to be emphasised is that the graph for adiabatic expansion is everywhere steeper than for isothermal expansion. Similarly if we compress a gas adiabatically the graph of p on a base V (Fig. 46) will everywhere be steeper than for isothermal compression. For in adiabatic compression, with no escape of heat, the gas will be warmed and the pressure will consequently rise more than in isothermal compression where heat must be in some way taken away from the gas to keep it at a constant temperature. Thus in adiabatic compression of air to half its initial volume, its pressure would be raised to 2·65 times its initial pressure against twice in the case of isothermal compression. We considered in Art. 55 why a gas was cooled by expansion. The reason why a gas is heated by compression is very similar, but as compression is so important in some heat engines we may well consider specifically the cause of heating by compression.

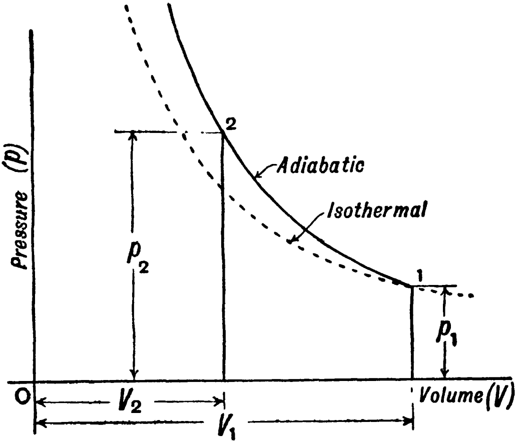

FIG. 46.—p-V diagram for adiabatic compression.

We have noted that a molecule of gas moving in a closed vessel with a velocity of which the component normal to a stationary side is v, after impact with the wall, rebounds with its normal component reversed in direction. Also that if it strikes a receding piston its normal velocity of rebound is less than v. But in compression, some of the molecules will meet an advancing piston (Fig. 47) and their normal velocity of impact relative to the piston will be greater than v. Consequently the normal component of the velocity of rebound will be increased and the actual velocity of the molecule, and therefore its kinetic energy, will be increased (see Art. 36). That is, the temperature of the gas will be raised unless by some means heat is withdrawn from the gas as in isothermal compression. To be more specific, if the velocity of the piston is v', the relative velocity at impact will be $v'+v$ and after impact $-(v+v')$; and the absolute (normal) velocity of the molecule will be $-(v+2v')$ or $v+2v'$ in a direction opposite to the original direction of v. The heating of air by compression is quickly made evident by the brisk use of a bicycle pump. If the pump is made of a poor conducting material the conditions may approach those of adiabatic compression, but with a metal pump, though more heat may be conducted away, its effect is more quickly noticeable by the sense of touch.

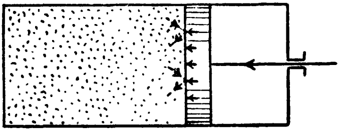

FIG. 47.—Molecular acceleration in compression.

The conditions of adiabatic compression may be approached by the use of materials which are bad conductors of heat, but they may also be approached in a metal pump or compressor by quick compression which allows little time for the flow away of any heat.

The relation between pressure and volume in adiabatic expansion is $p \times V^{1 \cdot 4} =$ constant for air, so that for Figs. 45 and 46, $p_1V_1^{1 \cdot 4}=p_2V_2^{1 \cdot 4}$. From these relations the reader may verify the pressure changes quoted at the beginning of this article for a change of volume of 2 to 1.

In general, expansion or compression may follow a law $p \times V^n$=constant, and for isothermal expansion $n=1$, while for adiabatic expansion n is equal to the ratio of the specific heat at constant pressure to that at constant volume which for air is approximately 1·4. The further development of this part of the subject is appropriate to a higher stage. But we may note that the steepness of the p-V curve is proportional to n. That of the adiabatic is 1·4 times that of the isothermal curve.

58. Energy Conversion and Principle of Added Heat.

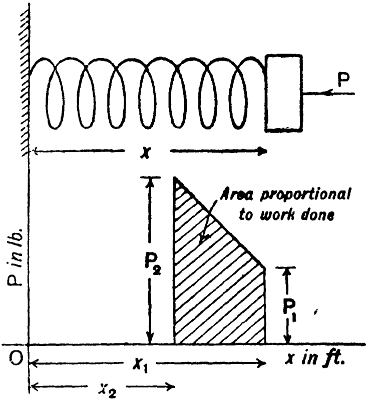

FIG. 48.—Work diagram in spring compression.

Mechanical energy is often stored, temporarily at least, by straining a spring as in taking up surplus energy to avoid shock, in the springs of motor vehicles, the shock absorbers of aeroplanes or in buffers. In this case the force P increases in proportion to the elastic strain of the spring (Fig. 48), that is, to the change in the distance x. The diagram of work is as shown in Fig. 48 and the energy stored is the average force multiplied by the displacement x_1-x_2, and the graph being a straight line the average force is $\frac{1}{2}(P_1+P_2)$, or the energy stored is

$$\tfrac{1}{2}(P_1+P_2) \text{ lb.} \times (x_1-x_2) \text{ ft.} = \tfrac{1}{2}(P_1+P_2)(x_1-x_2) \text{ ft.-lb.}$$

Now consider the case of compression of a gas, say air, in a cylinder. It also acts as a spring and may be used as a cushion to take up the energy of an impact, but we will consider compression in a cylinder by a piston and the energy stored, without regard to any particular purpose. In this case the force required for compression does not simply rise in proportion to the distance or diminution of volume of the air compressed, but as we have already seen, it rises in a curve the steepness of which increases. And if the compression is adiabatic, as it will be approximately if the compression takes place quickly, the curve is steeper than it would be if all the heat resulting from compression were continually conducted away so as to maintain isothermal conditions (say, temperature constant at t_1°). The p-V graph *ab* for adiabatic compression of a quantity of air at pressure p_1 lb. per sq. ft. and initial volume V_1 cu. ft., is shown in Fig. 49. Adiabatic compression in this case is to a volume V_2 at *b*, which is $\frac{1}{4}$ of V_1, and the pressure rises to p_2, which is nearly 7* times p_1 (instead of 4 times p_1, as it would be for isothermal compression). The work done in compression is shown by the shaded area *abde* under the adiabatic graph *ab*. This is the amount of mechanical energy spent by some outside source, and it is all converted into heat energy, i.e. it increases by this amount the kinetic energy of the flying molecules of the gas. The air is heated to, say, t_2° and gains energy $C_v \times w \times (t_2 - t_1)$ B.Th.U., where w is the weight of air in pounds, t_1 and t_2 being in degrees Fahrenheit. Now suppose the compressed air stands in the cylinder, the piston remaining in the same position, and the heat is conducted away till the temperature falls again from t_2° to the initial temperature t_1° (say, that of the atmosphere). All the energy previously added by compresssion goes out of the air, and its pressure falls at constant volume in proportion to the fall of temperature, to, say, p_c, so that $p_c/p_2 = \frac{t_1 + 460}{t_2 + 460}$ for Fahrenheit temperatures.

We have said that all the energy previously added has gone out of the gas, and yet there is at *c* the volume V_2 of gas at a pressure p_c, from which we should rightly

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[1] The numerical values of pressure changes and ratios of areas quoted in the article can be calculated from the relation $p.V^{1.4}$ =constant.

expect that mechanical energy could be obtained by allowing it to expand to a pressure p_1, that of the atmosphere.

The explanation of this seeming paradox is that if the air is allowed to expand adiabatically from the pressure

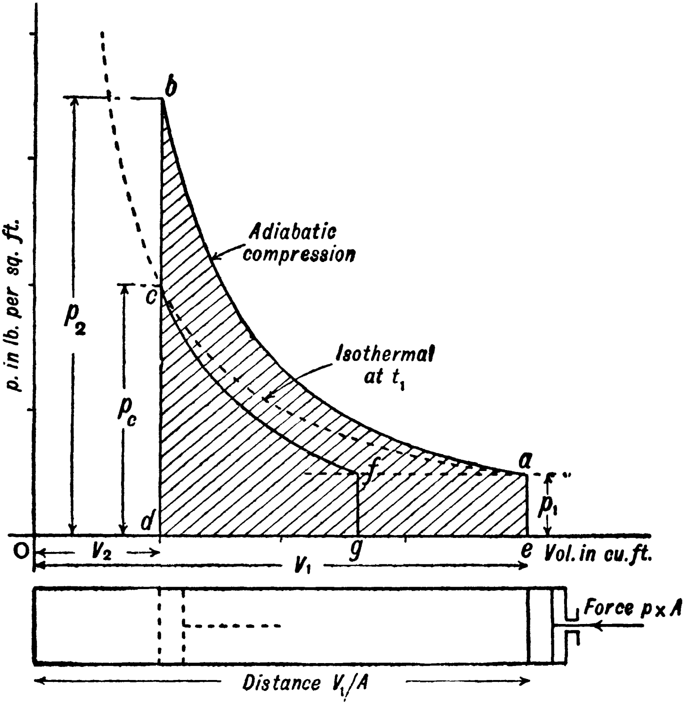

FIG. 49.—p-V diagram of adiabatic compression and cooling at constant volume.

p_c at c to the atmospheric pressure (equal to p_1) at f, doing work on the piston, it is cooled below the temperature of the atmosphere, t_1, and so, later, may draw on the stock of heat energy in the atmosphere to restore its temperature to t_1. In fact it will subsequently, if released, pick up this energy from the atmosphere by becoming warmed up to the temperature $t_1°$. The energy obtainable by adiabatic expansion in this way from *c* to atmospheric pressure p_1 at *f*, is the area under *cf*, i.e. the area *cfgd*, which represents about 44 per cent. of the energy added to the air by compression that is represented by the shaded area *abcde*.

The preceding paragraph on the recovery of energy from the cooled compressed air is a digression, but it serves to illustrate how different an air cushioning device is from an elastic steel spring and to satisfy a query that may naturally arise from the statement that the air after cooling has lost all the energy previously given to it. Moreover it contains the principle of an early form of refrigeration which consisted of compressing air, cooling it at high pressure, and then allowing it to expand and so do work and lose heat energy and become cooled below freezing point. We now return to the fact that under adiabatic conditions, all the energy given to the air in compression is recoverable in expansion, except for losses due to imperfection of the apparatus, and proceed to a point of great importance.

Suppose that at the end of compression along *ab*, instead of being cooled and thereby losing heat energy, the air receives heat energy before re-expansion ; both the pressure and the temperature will thereby be raised. Then *more* work will be done in a subsequent expansion to atmospheric pressure, or even to a pressure above the atmospheric, than was done in compression. We shall, in fact, have some heat energy supplied as heat to the air in the cylinder, converted into mechanical energy by the expansion of the air, doing work on the piston. Or in other words, the molecules will have their speeds increased by the addition of heat to the compressed air and during the expansion they will therefore inflict on the receding piston both greater and more frequent blows, and give up more energy than if they possessed only the speed acquired in the compression.

Air Engines. The attempt to use heat in this way was made by the help of various devices early in the nineteenth century but always attempting to give the heat to the air by contact with hot bodies, that is, by *external* heating. These *air engines*, which might be classed as *external* combustion engines, were inefficient because air is a poor conductor of heat; heating by conduction is slow and large surfaces are required to effect it. Only small rises in temperature and pressure were found practicable and air engines were very large and heavy in relation to the power they developed, and much energy was wasted in overcoming friction in the necessarily large cylinders.

59. Internal Combustion Engines. Although heating air after compression by contact with externally heated hot bodies proved largely ineffective, the problem of heating air quickly and to a high temperature and pressure has been solved by *internal combustion*, that is, by burning a suitable fuel which provides the heat, within the cylinder.

The internal combustion engine is essentially an *air* engine. For in it charges, mainly of air, are compressed and then the temperature and pressures are augmented (in many cases largely) by the combustion of a gas, vapour or oil for which the necessary oxygen is provided by the compressed air. Work is then done on the piston by the expansion of the working gases which consist of (1) the gaseous products of combustion; (2) any surplus oxygen from the charge of air remaining in the gases after combustion, but mainly (3) the inert gas nitrogen which constitutes about four-fifths by volume of the atmosphere at the level of the earth's surface. The greater part of the air charge which is compressed is, of course, nitrogen, for the charge is air together with any small amount of exhaust gases which have remained from a previous expansion stroke of the engine. Thus the working fluid is essentially air in which the element nitrogen, taking no active part in the combustion, acts as a dilutant and prevents the temperatures rising as high as it would without the presence of this inert gas. In this respect the internal combusion engine differs widely from a gun in which the working fluid which propels the projectile consists of the gaseous products of a combustion (the oxygen for which is supplied, not by the air, but by solid substances which form part of the charge) and not mainly of nitrogen.

The foregoing paragraphs have been concerned with the fundamental principles of the internal combustion engine, but this is only a small part of the subject. The cycles of operation, the manner of igniting and burning, the various liquid and gaseous fuels, the mechanical devices by which the air is forced into the cylinder and by which the gaseous products of combustion are expelled, by which the piston and valves are made gas-tight and the means of keeping the cylinder cool enough to allow of lubrication, constitute a very large field of technical knowledge, of which, at this stage, we can touch but a very small part. But in all cases the essential feature is that the molecules of the complex mixture of gases, but mainly nitrogen, derived from the atmosphere, attain a high velocity and therefore kinetic energy, converted from chemical energy by combustion, impinge on a piston on which they do work and so give up part of their kinetic energy and are therefore cooled during their expansion.

This cooling of a gas in expansion and heating by compression may well excite our wonder, so quickly does it take place in comparison with the speed of transfer of heat by conduction. An engine may have its working gases cooled through several hundred degrees in, say, one-fortieth of a second. We have considered mainly adiabatic expansion as with non-conducting materials but actually expansion takes place in metal cylinders which may exert some influences on the temperature of the gas, but that influence is minimised by the speed at which the engine operates.

As already stated, the temperature and pressure are augmented by the combustion of the gas, oil, or vapour and this often takes place at approximately constant volume. But in other cases when compression is to a high pressure, the object is not to raise the gas to a higher temperature and pressure which would be difficult to deal with for reasons of strength and lubrication, but to maintain the pressure for a part of the time during which the volume is increasing and work is being done on the piston. We shall not go into the different types, but we may clarify the principle explained above by reference to the ideal p-V or work diagram for the case in which the heat energy given out in combustion is given to the compressed air at constant volume. This is shown in Fig. 50, in which p is the pressure in pounds per square foot and V is the volume of air or gases in cubic feet.

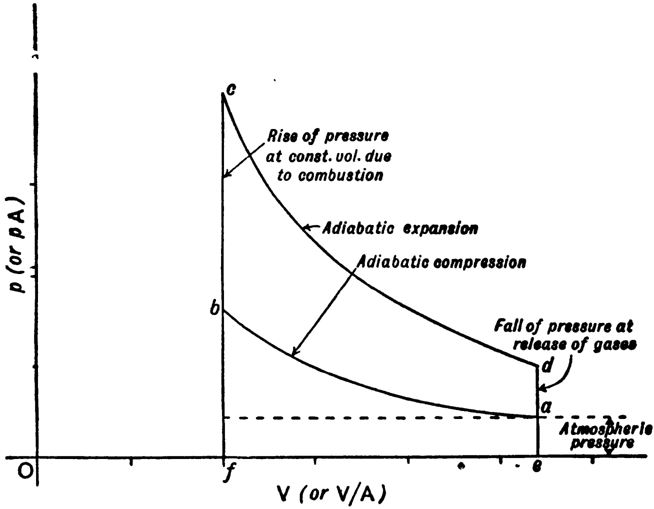

FIG. 50.—p-V diagram of principle of internal combustion engine.

Areas will then show work in foot-pounds on a scale such that 1 sq. in. represents as many foot-pounds as the product of the number of pounds per square foot represented by 1 in. and the number of cubic feet represented by 1 in. (Alternatively, to a different scale, if A is the area of piston in square feet, the vertical ordinates represent $p \times \mathrm{A}$, the total force of the gas on the piston, and the horizontal distances represent $\frac{\mathrm{V}}{\mathrm{A}}$, the distance moved by the piston in feet), where V is the excess of volume over that at the end of the stroke (called the clearance volume). Then in Fig. 50, *ab* represents the adiabatic compression of air from atmospheric pressure to a pressure represented by *fb*, the work done in compression being represented by the area under *ab*, namely *abfe*. And *bc* represents the rise of pressure due to combustion at the constant volume O*f*. The curve *cd* represents the adiabatic expansion of the high pressure gases (mainly nitrogen) to the original volume O*e* doing work on the piston represented by the area *cdef* under the expansion curve, while the vertical line *da* represents the fall of pressure to that of the atmosphere on the release of the gases at the end of the expansion, that is, at the opening of an exhaust valve. The net work done in such a cycle of operations would be represented by area *cdef* minus area *abfe* (the work done in compression) or the net work is represented by the shaded area *abcd*. The following examples are to illustrate numerically the foregoing principle. In order to simplify them a constant specific heat at constant volume is assumed although, at the temperatures and pressures in actual internal combustion engines, the specific heat is not constant nor is the composition of the working fluid, which, after ignition, contains the gaseous products of combustion.

Example 1. 1 lb. of air at 14·7 lb. per sq. in. and 100° F. is adiabatically compressed to one quarter of its volume (following the law $pV^{1\cdot4}=\text{constant}$) which raises its pressure to $4^{1\cdot4}$, or 6·97, times its original pressure, that is, to 102·3 lb. per sq. in. Assuming that it behaves as an ideal gas, find its temperature and hence find in foot-pounds the work done in compression, taking the specific heat at constant volume as 0·1691.

Using Fig. 50, and supposing volume at $b=\frac{1}{4}$ of volume at *a*,

$$\frac{p_bV_b}{T_b}=\frac{p_aV_a}{T_a} \text{ or } \frac{T_b}{T_a}=\frac{p_b}{p_a}\cdot\frac{V_b}{V_a}=6\cdot97\times\tfrac{1}{4}=1\cdot74.$$

And $T_a = (460+100)°$ F. $= 560°$ F. absolute, hence,

$$T_b = 1{\cdot}74 T_a = 1{\cdot}74 \times 560° \text{ F.} = 974{\cdot}7° \text{ F. absolute}$$

Rise of temperature $T_b - T_a = 974{\cdot}7 - 560 = 414{\cdot}7°$ F.

Gain of internal energy of 1 lb. $= 1$ lb. $\times 414{\cdot}7°$ F. $\times 0{\cdot}1691$
$= 70{\cdot}13$ B.Th.U.

And since the compression is adiabatic, the only gain of internal energy is due to the work done in compression and its heat equivalent is 70·13 B.Th.U., hence

Work done $= 70{\cdot}13$ B.Th.U. $\times 778$ ft.-lb./B.Th.U.
$= 54{,}560$ ft.-lb.

Example 2. Suppose the 1 lb. of air in Example 1, after adiabatic compression, is heated at constant volume by 500° F. increase of temperature, at constant volume, and is then adiabatically expanded (fourfold) to its original volume, its pressure consequently being reduced in the ratio 6·97 to 1. Assuming its specific heat at constant volume (0·1691) remains constant, find:

(*a*) Its pressure after heating at constant volume;

(*b*) The gain of energy in B.Th.U. after heating;

(*c*) The pressure and temperature after adiabatic expansion;

(*d*) The work done in foot-pounds in adiabatic expansion;

(*e*) The B.Th.U. equivalent to the excess of work done in expansion over that spent in compression;

(*f*) The ratio of this excess to the heat supplied or gain of energy by heating at constant volume.

Again referring to Fig. 50, heating at constant volume being from *b* to *c* and adiabatic expansion from *c* to *d*:

(*a*) For heating at constant volume the pressure is proportional to the absolute temperature and from Example 1, the absolute temperature at *b* was $T_b = 974{\cdot}7°$ F. and with 500° added $T_c = 974{\cdot}7 + 500 = 1475°$ F., say. Hence, pressure at *c*

$$p_c = p_b \times \tfrac{1475}{975} = 102{\cdot}3 \times \tfrac{1475}{975} = 154{\cdot}8 \text{ lb. per sq. in.}$$

(*b*) Gain of energy in B.Th.U. $= 1$ lb. $\times 0{\cdot}1691 \times 500°$ F. $= 84{\cdot}5$ B.Th.U.

(*c*) After adiabatic expansion, the pressure falls to $\frac{154 \cdot 8}{6 \cdot 97} = 22 \cdot 2$ lb. per sq. in.

And since $\frac{p_c V_c}{T_c} = \frac{p_d V_d}{T_d}$,

$$T_d = T_c \times \frac{p_d}{p_c} \times \frac{V_d}{V_c} = 1475 \times \frac{1}{6 \cdot 97} \times \frac{4}{1} = 860° \text{ F. abs.}$$

a drop of 1475° − 860° = 595° F.

(*d*) The loss of internal energy reckoned from the cooling in adiabatic expansion is,

$$1 \text{ lb.} \times 0 \cdot 1691 \times 595° = 100 \cdot 8 \text{ B.Th.U.}$$

and since the work done is at the expense of the internal energy, this is the heat equivalent of the work done, and,

$$\text{Work done} = 100 \cdot 8 \times 778 = 78{,}422 \text{ ft.-lb.}$$

(*e*) The heat equivalent of the excess work in expansion over that in compression is

$$100 \cdot 8 - 70 \cdot 1 = 30 \cdot 7 \text{ B.Th.U.}$$

(*f*) Ratio of above excess to the heat supplied at constant volume is

$$\frac{30 \cdot 7}{84 \cdot 5} = 0 \cdot 363 \text{ or } 36 \cdot 3 \text{ per cent.}$$

For a higher rise of temperature and pressure from *b* to *c* such as actually occurs in internal combustion engines, the above ratio would be higher, although the assumption of a constant specific heat would then be less reasonable, while a variation in specific heat would greatly complicate the numerical illustration of the principle of an internal combustion engine.

60. Note on Ideal Pressure-Volume Diagrams. The foregoing pressure-volume diagrams (Figs. 49 and 50) and others to follow, are ideal and the pressure in actual engines does not follow the pressure of the ideal process in various respects. In the main, it is less, and a true pressure-volume diagram would have its corners rounded off, particularly at *d* and *a*, for it is not possible to get gas out of or into a cylinder instantaneously. However large the valve passages are made, there is some obstruction and delay, and the faster an engine runs, the greater the effect on pressure, of the failure to transfer the working fluid with sufficient speed from place to place. Indicators are used to show the relation between the pressure and the position of the piston, and indicator diagrams show in some measure the deviation of the pressure-volume graph from the ideal. But indicators themselves introduce errors and do not show the true pressure-volume graph. Consequently we have referred to the deviation of the actual pressure from that shown in the ideal pressure-volume diagram rather than to the deviation of the indicated pressure from that in the ideal pressure-volume diagram.

61. Compression and Expansion of Vapours. We have seen that the work done in compression of a perfect gas produces a heating effect and, unless heat is disposed of, a rise of temperature. And that expansion produces a cooling effect, and the temperature falls unless heat is supplied to maintain it. And the same is true of actual gases. Furthermore, a vapour such as steam, if superheated, behaves in a similar way, even if near the temperature of liquefaction, provided that it remains just superheated. But if the vapour, say steam, is saturated or actually a little wet, that is, carrying unevaporated particles of its liquid (water) in suspension, more or less like a mist or fog, the effect of compression or expansion is somewhat different from that on a gas. The increased heat energy equivalent to the work done in compression, if there is no escape of heat, goes partly to raise the temperature of the saturated vapour, but so long as the steam remains "wet" this increase of heat energy instead of serving to superheat the steam, evaporates some of the water particles present. Just so much water is evaporated as will keep the steam saturated. Thus wet steam, carrying a little water in suspension is made drier by compression under adiabatic conditions. Similarly the cooling of the saturated or wet vapour by adiabatic expansion serves to condense some of it, producing water either as fine mist particles or as a film on the walls of the cylinder or other containing vessel, or as both. Thus dry saturated steam becomes wet and wet steam becomes wetter by adiabatic expansion. Except at very high pressures, the volume occupied by such water of condensation is negligible in comparison with the very much larger volume of the steam and therefore, during expansion there is in effect a diminishing amount (weight) of saturated steam, together with some water of negligible volume, and during compression there is an increasing amount of steam augmented by the evaporation from the water present.

Example. 1 lb. of wet steam at 15 lb. per sq. in. pressure is adiabatically compressed to a pressure of 80 lb. per sq. in. If initially the mixture was 20 per cent. water and 80 per cent. saturated steam (i.e. having a dryness fraction of 80 per cent.) and the final dryness is 86·5 per cent., find how many foot-pounds of work have been done in the compression.

From the steam table, after compression—
Sensible heat per pound at 80 lb./sq. in = 281·9 B.Th.U.
Latent heat $=0{\cdot}865\times904{\cdot}2$ = 782·1 B.Th.U.

Total heat in 1 lb. of wet steam =1064·0 B.Th.U.

Before compression, from tables—
Sensible heat per pound, at 15 lb./sq. in. = 181·0 B.Th.U.
Latent heat $=0{\cdot}8\times970{\cdot}2$ = 776·2 B.Th.U.

Total heat of wet steam per pound = 957·2 B.Th.U.

Heat energy gained in compression, $=1064{\cdot}0-957{\cdot}2$ = 106·8 B.Th.U.

Work in compression to produce this $=778\times106{\cdot}8$ =83,080 ft.-lb.

62. Work done in Production of Steam. If steam were produced inside a cylinder by heating water until it evaporated, the steam would occupy much more room than the water from which it was formed, and in order to make room for itself, the steam during formation would have to push back the atmosphere or whatever restricted its expansion and would so do work. Consider a simple example. Suppose 1 lb. of water in a cylinder (Fig. 51) say, 1 sq. ft. in cross-sectional area and loaded through the medium of a frictionless piston with a force (including the pressure of the atmosphere) of 40 lb. per sq. in. or 40 × 144 lb. per sq. ft. If heat is supplied just sufficient to evaporate all the water to dry saturated steam, the volume occupied, as shown by the steam table, will be 10·5 cu. ft. (the specific volume of steam at 40 lb. per sq. in.). The piston will have moved (upwards in Fig. 51) 10·5 ft. since the area of piston is 1 sq. ft., and the work done will be 40 lb. per sq. in. ×144 sq. in. ×10·5 ft. =60,480 ft.-lb., equivalent to $\frac{60,480}{778}$ B.Th.U. =77·7 B.Th.U.

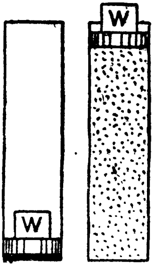

FIG. 51.—Work done in producing steam.

This work done during evaporation represents part of the energy, supplied as heat, to evaporate the water and called the latent heat of evaporation, L, which is at this pressure 935·6 B.Th.U. per lb.

The remainder of the heat supplied, namely,

$$935·6 - 77·7 = 857·9 \text{ B.Th.U.}$$

also becomes latent and goes to give the molecules the kinetic energy of rapid motion which they possess in the vapour, but not in the liquid. This latter part of the energy necessary to evaporate the liquid is called the increase of *internal energy* of the steam. In this example 77·7/935·6 or about 8·3 per cent. of the heat of evaporation is spent in external work, the other 91·7 per cent. being increase in internal energy, i.e. increase in kinetic energy of the molecules.

It will be noticed that the work done in evaporation is similar to the work done in heating a gas at constant pressure (Art. 55) except that the initial volume is so small as to be negligible. In the numerical case chosen above, the volume occupied by the water is about 0·017 cu. ft. (or a height of 0·017 ft. in the cylinder of Fig. 51, which is negligible in comparison with the 10·5 cu. ft. (or 10·5 ft. travel of piston), being only about one-sixth of one per cent. (It would only reach about 1 per cent. in steam at as high a pressure as 270 lb. per sq. in.) Taking the general case, if V is the volume of steam per pound (called the specific volume of steam) at a pressure of p lb. per sq. ft. the work done in evaporation, neglecting the volume of water from which it is formed, is

$$p \text{ lb./sq. ft.} \times \text{V cu. ft.} = p\text{V ft.-lb.}$$

$$\text{and L} = \left(\frac{p\text{V}}{778} + \text{increase of internal energy}\right) \text{B.Th.U.}$$

Steam as a Working Fluid. Steam evaporated from water at temperatures above that of the atmosphere and having a high critical temperature (above which it can only exist as a vapour or gas) obviously differs widely from air with critical temperature much below that of the atmosphere. Steam has a high latent heat and therefore it can carry and transmit to the cylinder of an engine a larger amount of energy, even at moderate temperatures and pressures. And the part of this, pV/778 B.Th.U., which represents external work done in evaporation, through a small fraction of the latent heat, is in fact large. In other words steam at any given pressure has a relatively large specific volume or volume per pound and these facts have a bearing on the usefulness of steam as a working fluid. We are not in a position to discuss fully the complicated question of the suitability of working fluids which are affected by many factors, but we may notice some difference between air and steam at relatively low pressures. If 1 lb. of air at atmospheric pressure were compressed adiabatically to one quarter of its volume, its pressure would rise to about 102 lb. per sq. in. and its temperature from, say, 60° F. to about 446° F., and it would occupy about 3·1 cu. ft. At the same pressure, 1 lb. of dry saturated steam would occupy about 4·37 cu. ft., and in addition to the sensible heat in the water from which it was evaporated, would have a latent heat of about 892 B.Th.U. of which about 83 B.Th.U. would represent the external work done at constant pressure in evaporation and its temperature would be only about 329° F. To give the 1 lb. of air after compression to this pressure an amount of energy equal to 83 B.Th.U. would involve heating it through a further temperature of nearly 500° F. to a temperature of over 900° F. And to give it, after compression, an increase of energy equal to the increase in energy of the 1 lb. of steam on evaporation, would involve heating it by 4000° or 5000° F. (It is not easy to give precise figures because the specific heat of air rises considerably at higher temperature.) The reader is warned against drawing the conclusion that because steam can carry so much energy into a cylinder, it can convert any correspondingly high amount of heat energy into work. We here only make the point that steam can carry much energy at relatively low temperatures and low pressures. It also has the manifest advantages of abundance and low cost, and of being condensable at low pressure by natural cooling waters such as rivers, lakes and seas.

63. Principles of the Steam Engine. The principle of operation of the steam engine is in some ways like that of the internal combustion engine. The expansion of a vapour (steam) in a cylinder is used to extract from it some of the internal heat energy, that is, some of the kinetic energy of its molecules, which impinge upon, and do work upon, the receding piston, and are thereby cooled. For, as with a gas, they rebound from the receding piston with diminished speed. The expansion differs from that in an internal combustion engine in the fact that the vapour becomes wet, i.e. some of it is condensed to water (as explained in Art. 61) unless the steam is initially so much superheated as to remain above saturation point during the cooling which results from the loss of internal energy equivalent to the work done in expansion. But in two other important respects the operation differs from that of an internal combustion engine. Firstly, the expansion at constant pressure, from water to steam, doing work as explained in the preceding article (Art. 62). This is actually carried out not in the cylinder but in a separate organ, the boiler. But when a valve is open between the boiler and the cylinder, it is the movement of the piston which provides for the expansion of volume at constant pressure and which so allows the steam to do work (pV) during evaporation. The intervening steam transmits force and energy much in the same way as water under pressure (Chap. IV). Thus the effect is the same as if the evaporation were in the cylinder. The fact that it is actually in a boiler heated by external means, marks the steam engine as one of external combustion in contrast to the *internal* combustion in gas, oil, and spirit engines. Secondly, *condensing* steam engines also differ from internal combustion engines in employing a partial vacuum, i.e. a pressure below that of the atmosphere, to assist the piston on its return stroke, after it has done work by the admission of steam from the boiler to the cylinder. In a single-acting engine (Fig. 52), the force driving the piston (and so doing work) on the backward stroke may be the difference between the pressure of the atmosphere on one side of the piston (the open end of the cylinder, Fig. 52) and the lower pressure on the other side due to cooling and condensing the used steam in a separate organ called the condenser. But in a double-acting engine (Fig. 53) the return stroke is accomplished by steam pressure acting on one side of the piston against

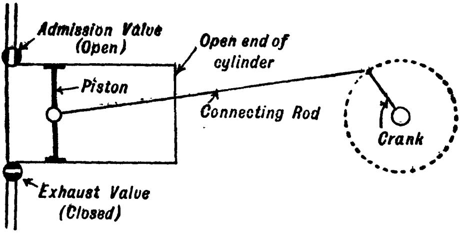

FIG. 52.—Diagram of single-acting steam engine.

the pressure of the atmosphere on the other side, in a non-condensing engine, or against a pressure brought

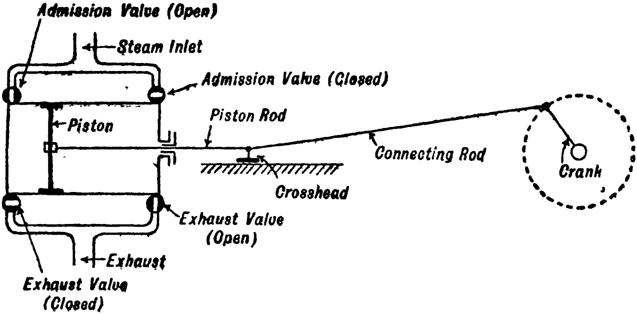

FIG. 53.—Diagram of double-acting steam engine.

below that of the atmosphere by the action of the condenser, if the engine is of the condensing type.

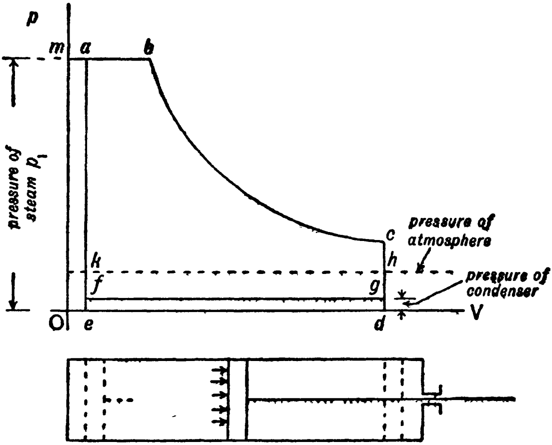

FIG. 54.—p-V diagram for forward stroke.

The action of the steam on the piston may be made clear by the use of the ideal p-V or work diagram shown in Fig. 54. which relates to one side of the piston only, From *a* to *b* a volume of steam represented by *ma*, (the clearance space at the end of the cylinder and passages) is increased at constant pressure p_1 by the admission of additional steam, that is, virtually by the evaporation of an equal weight of water, at the boiler pressure p_1. At *b* the supply of steam is cut off by a valve and expansion takes place. The expansion is ideally adiabatic, thereby making the saturated steam increasingly wetter or making superheated steam less superheated. At *c*, the piston having reached the end of its stroke, an exhaust valve allows the steam to escape to the atmosphere or to a condenser, and the pressure falls in the first case to *h* or in the latter to *g*, and the piston returns. The work done *by the steam pressure on the piston* is represented by the area *eabcd*. Note that some of this work will be spent in overcoming through the stroke, whatever pressure, atmospheric or less, is exerted on the other side of the piston according as the engine has not, or has, a condenser.

If the engine is single-acting, that is, if steam is admitted to one side only of the piston, as in Fig. 52, the force urging the piston along the back stroke will be that of the atmospheric pressure from the open end of the cylinder, and the work done by this will be represented by the area *hdek*, all of which will be used in overcoming an equal back pressure at the closed end of the cylinder when the exhaust valve is open, if the engine is non-condensing.

In other words, since there is no excess of force on one side of the piston to drive it back, in order to overcome any friction and to accelerate the piston, a supply of energy must be drawn from the kinetic energy in the other moving parts of the engine, notably from the flywheel. If, however, the single-acting engine is of the condensing type, pressure urging the piston along the back stroke will be positive, namely, the excess of the pressure of the atmosphere for the open end of the cylinder, over that of the condenser at the closed end when the exhaust valve is open. And the work done *by the atmosphere* on the piston will be represented by the area *hdek*, of which the part *gdef* will be spent in overcoming the pressure of the condenser in the cylinder.

If the engine is double-acting (Fig. 53), the force urging the piston along the back stroke will be the steam pressure admitted to the right hand or opposite side of the piston to that shown in Fig. 54 and part of it will be used in overcoming the pressure of the atmosphere on the left side of the piston if the engine is non-condensing, or in overcoming the smaller pressure of the condenser if the engine is of the condensing type. Fig. 55 shows the *p*-V or work diagram for the back stroke.

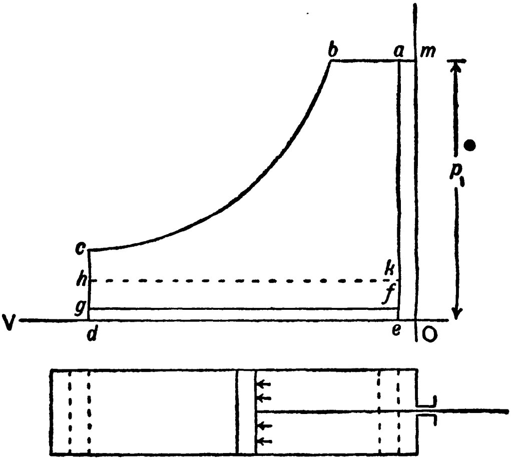

FIG. 55.—*p*-V diagram for back stroke.

In all cases there will be some clearance volume at the ends of the cylinder (including steam passages), and the work done by the steam on the piston relates only to the portion of the *p*-V diagram corresponding to the motion of the piston on which the steam molecules do work by their impact, as it recedes. The working volume, or volume swept through by the piston, is equal to the product of the area of the piston multiplied by the length of the stroke, but the volume of steam present exceeds this by the volume of the clearance, including the steam passages.

64. Actual Conditions in Steam Engines. In the preceding article the principles of the steam engine have been explained without reference to effects and defects necessarily arising to a greater or less extent in all actual engines. In real engines of the reciprocating type, that is, with a piston moving to and fro in a cylinder, valves and pistons are generally not absolutely steam tight and some leakage occurs while the engine is working. In addition to fluid back-pressures, the piston offers some frictional resistance to motion in the cylinder. And the materials of which cylinders and pistons are made are usually metallic and far from being non-conducting. Consequently the conditions of adiabatic working are not fulfilled and the heating and cooling of the cylinder walls and piston, storing and then giving out heat, may play an important part in the working of the engine.

The metal cylinder walls having just been in contact with the relatively cool exhaust steam, may at the admission of fresh and hotter steam condense some of it to form water on the walls. And during the exhaust period the reduced pressure may result in the re-evaporation of some of this water, thus taking heat (notably the latent heat) from the walls and so effectually cooling them quickly. Thus some heat energy is withheld by the metal from its opportunity of conversion into mechanical energy and it passes away in the exhaust as heat. This unfortunate effect is a consequence of the reciprocating action of the piston which is intermittent in character and is avoided in the rotary form of steam engine called a turbine, in which every part runs at approximately constant temperature.

Indicator Diagrams. An engine indicator is an instrument designed to produce automatically a *p*-V diagram showing the relation between the pressure at any instant in the engine cylinder and the corresponding position of the piston (and therefore the volume swept through from the beginning of the stroke up to that position). In order to do this, some form of tracing point is moved in one direction by amounts proportional to pressure changes in the cylinder; and in another direction at right angles to the first, it is given motion relative to a paper "card" or other background proportional to the movement of the piston. In early large slow-moving steam engines this was accomplished by simple mechanical devices, a steel spring giving the primary motion which varied with pressure charges transmitted from a small piston. As steam and later internal combustion engine practice developed, engines ran at increasingly high speeds and indicators were made lighter to avoid errors due to the force absorbed in accelerating so highly their moving parts and ultimately, for very high speeds, electrical indicators have been adopted. The types of indicators and their mechanisms are so numerous that there is no opportunity to enter upon a study of them here. An indicator should be examined and used in the laboratory. There are two main uses of an indicator diagram : (1) to show the pressure changes at all times in the cylinder and so examine how the valves are working and whether any change is required ; and (2) to measure the work done by the working fluid on the piston. Assuming that the indicator successfully performs its purpose we find certain characteristic variations of the indicator diagram from the ideal p-V or work diagram, sometimes rather unhappily called the "hypothetical indicator diagram." We can best illustrate the variations by an example. Fig 56 shows a typical indicator diagram for one end of the cylinder of a condensing engine. (1) Steam is admitted and reaches the pressure shown at a but somewhat below the boiler pressure, due to restriction of the flow in the pipes, valve, and passages. (2) As the piston moves with increasing speed the pressure falls somewhat along the steam line ab. (3) About b the admission valve closes, but closure takes appreciable time during which the steam has to pass through a restricted opening and is throttled and pressure begins to fall before the complete closure at b, and the diagram is thus rounded off at b and not sharp as in the ideal p-V diagram. (4) The expansion curve bc is not necessarily adiabatic nor hyperbolic but depends upon whether the metal cylinder and piston are taking heat from the steam or giving heat to the steam and upon possible leakage. (5) At *c* the exhaust valve opens a little before the end of the stroke in order that the pressure may drop before the return stroke begins, but the time taken in opening the valve results in a gradual drop in pressure and rounding off what is called the toe of the diagram. (6) At *d*, near the beginning of the back stroke, the back pressure is down to its lowest level but never quite so low as in the condenser. (7) At

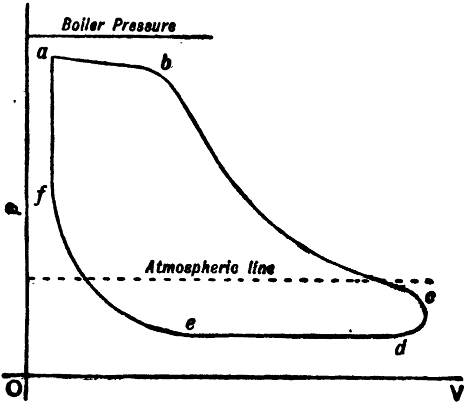

Fig. 56.—Indicator diagram.

e the exhaust valve closes, and the remaining steam shut in is compressed and dried somewhat before the admission valve opens about the point *f*. This compression has the mechanical "cushioning" or shock-absorbing effect of taking up some of the energy of the piston and other reciprocating parts as they come to rest at the end of the stroke.

The calculation of the work done per stroke, and hence the indicated horse-power, from the indicator diagram, has been dealt with in a previous course [1] but we may recall that if p is the mean effective pressure per square inch, A the area of the piston in square inches, and N working strokes of L ft. are made per minute, then

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[1] *Elementary Engineering Science* (Morley and Hughes), Art. 29.

The average force on the piston is

p lb./sq. in. × A sq. in. $=pA$ lb.

The work per stroke is pA lb. × L ft. $=pAL$ ft.-lb.

And the work per minute is $pAL \times N$ ft.-lb. per min., and since 33,000 ft.-lb. per min. is 1 h.p.

$$\text{i.h.p.}=\frac{p\times A\times L\times N}{33{,}000}$$

The Ratio of Expansion. If the steam supply is cut off when the piston has travelled, say, one third of its stroke, then, excluding the clearance, the final volume is 3 times that at the cut-off and the *nominal* ratio of expansion is 3. And in general, the nominal ratio of expansion r is the ratio of the full volume swept through by the piston to that swept through from the beginning of the stroke to the position at which the steam supply is cut off. The actual ratio of expansion is somewhat smaller because both at cut-off and at the end of the stroke the volume of steam in the cylinder is greater than the nominal by the amount in the clearance (including the steam passages).

Since the work done on the piston by the steam is at the expense of its internal energy it might appear desirable to make the ratio of expansion very large and so get more work from the steam. But there are severe limitations to this. What is required is not to be measured by work done by the steam on the piston, but by work done by the engine crankshaft in accomplishing some desired object. It is obviously useless to continue the expansion to a degree at which the forward thrust of the expanding steam is doing no more than overcome the back pressure on the opposite side of the piston and the frictional resistances of the engine. Actually it is not worth while carrying it beyond such a degree that the steam pressure is sufficiently in excess of the resistance to be doing a substantial amount of useful work. For the greater the expansion the larger is the cylinder required to accommodate the final volume of steam at the low pressure. The initial volume of, say, a pound of steam is fixed by the boiler pressure and the final volume depends upon the size of the cylinder. And larger cylinders involve not only more friction but also a more costly engine. Hence it comes about that in the simple single cylinder engine the (nominal) ratio of expansion seldom exceeds about 3 (i.e. a cut-off not earlier than 1/3 stroke). There are additional reasons for limitation with a single cylinder which concern the degree of condensation within the cylinder, but these are for later study.

In order to estimate the size of cylinder required for any particular horse-power, the mean effective pressure cannot be calculated in advance, even for a known boiler pressure and ratio of expansion. For the actual pressures will, for the reasons explained above, not be the same as in any ideal p-V work diagram, and it is necessary to rely on previous experience and to have regard to the conditions such as the engine speed and the type of valves. In many cases the indicator diagram shows an expansion curve not greatly differing from the rectangular hyperbola which applies (see Art. 52) to the expansion of the ideal gas at constant temperature. A calculation based on " hyberbolic expansion " is often made the basis of the estimate of expected mean pressure. This is rather unfortunate, as it is apt to lead to the entirely erroneous conclusion that the expansion of saturated steam in a cylinder is related to the isothermal expansion of an ideal gas. Moreover the character of the mathematical calculation, generally carried to a considerable degree of precision is not justified when it has to be overridden by a factor based on practical experience which it is hoped may be correct within 5 per cent. It can be shown that the so-called hypothetical pressure based on an ideal p-V work diagram with hyperbolic expansion (i.e. pressure varying inversely as volume) is,

$$\text{hypothetical mean effective pressure}$$
$$=\text{initial pressure}\left(\frac{1+\log_e r}{r}\right)-\text{back pressure}$$

where $\log_e r$ is the hyperbolic or Naperian logarithm of r, which is given in hyperbolic or Naperian logarithm tables or may be found by multiplying the common logarithm by 2·3026.

The values of the factor $(1+\log_e r)/r$ are as follows :—

r	1	2	3	4	5
$\frac{(1+\log_e r)}{r}$ . . .	1	0·85	0·70	0·60	0·52

and for intermediate values of r can be found with sufficient accuracy by taking proportional parts.

If r does not exceed about 3·5, a value near enough can be found by taking $(1{\cdot}15-0{\cdot}15r)$ instead of $(1+\log_e r)/r$.

To obtain an estimate of the probable mean effective pressure the above hypothetical mean effective pressure is multiplied by a factor called a diagram factor which commonly varies from 0·6 to 0·9 according to circumstances. The diagram factor is, of course, the ratio of the area of the probable indicator diagram to that of the hypothetical p-V diagram with hyperbolic expansion. For the length of diagram is proportional to the *working* volume ; in both cases the breadths are proportional to the mean pressures and the areas are also proportional to the mean pressures.

In arriving at the dimensions of a steam engine for a given performance another item which is often decided from experience is the average piston speed which is generally specified as "piston speed" and generally expressed in feet per minute.

Example 1. If a mean effective pressure of 50 lb. per sq. in. may be expected and an average piston speed of 400 ft. per min. is allowed, find the approximate length of stroke, diameter of cylinder and revolutions per minute for a double-acting steam engine to develop 40 i.h.p. if the stroke is to be 1·4 times the diameter of the cylinder.

Let d inches be the diameter of the cylinder

Area of piston $= 0{\cdot}7854d^2$ sq. in.

Force on piston $= 0{\cdot}7854d^2$ sq. in. $\times 50$ lb./sq. in.
$= 39{\cdot}27d^2$ lb.

Work done per min. $= 39{\cdot}27d^2$ lb. $\times 400$ ft./min.
$= 15{,}708d^2$ ft.-lb./min.

But this is to give 40 i.h.p., hence,

$$15{,}708d^2 \text{ ft.-lb./min.} = 40 \times 33{,}000 \text{ ft.-lb./min.}$$
$$d^2 = 84$$
$$d = 9{\cdot}2$$

Thus the diameter is to be about 9·2 inches and the stroke $9{\cdot}2 \times 1{\cdot}4 = 13$ inches. In one rev., two strokes cover 26 in. $= 2{\cdot}17$ ft.

$$\text{Speed of rnnning} = \frac{400 \text{ ft./min.}}{2{\cdot}17 \text{ ft./rev.}} = 185 \text{ r.p.m.}$$

Example 2. For an initial gauge pressure of 80 lb. per sq. in. and a back pressure of 3 lb. per sq. in. above atmospheric pressure, and a cut off at 40 per cent. of the stroke, find the hypothetical m.e.p. (mean effective pressure) and the probable m.e.p. based on a diagram factor of 0·75.

$$\text{Initial pressure} = 80 + 14{\cdot}7 = 94{\cdot}7 \text{ lb./sq. in. absolute}$$
$$\text{Back pressure} = 3 + 14{\cdot}7 = 17{\cdot}7 \text{ lb./sq. in. absolute}$$
$$\text{Nominal Ratio of expansion} = \frac{100}{40} = 2{\cdot}5$$
$$\text{Hypothetical m.e.p.} = 94{\cdot}7\left(\frac{1 + 2{\cdot}3026 \log 2{\cdot}5}{2{\cdot}5}\right) - 17{\cdot}7$$
$$= 94{\cdot}7 \times \frac{1 + 2{\cdot}3026 \times 0{\cdot}3979}{2{\cdot}5} - 17{\cdot}7$$
$$= 54{\cdot}9 \text{ lb./sq. in.}$$
$$\text{Probable m.e.p.} = 0{\cdot}75 \times 54{\cdot}9 = 41 \text{ lb./sq. in.}$$

65. Supply and Disposal of Energy. It has been pointed out in Art. 62 that owing to its high latent heat of vaporisation, steam can, before entering an engine cylinder, take up a large amount of energy without being heated to an excessive temperature and pressure. Neglecting frictional losses and heat energy lost by conduction and radiation and leakages of steam, the energy spent in external work done in evaporation at constant pressure can be converted into mechanical energy. This is represented by the work done during the admission of steam to the cylinder, i.e. before the "cut off." And some of the increase of internal energy of the steam can also be used to do work on the piston during expansion. Nevertheless the total heat energy converted into mechanical energy is but a small fraction of that used in producing the steam. It is important to realise that much the greater part of the energy passes into the atmosphere or the condenser, according as the engine is non-condensing or condensing, in the form of latent heat of the exhaust steam. This steam may be wet and so not carry the full total heat of dry steam, but it is to be remembered that the latent heat per pound of dry saturated steam is higher at the lower temperatures and pressures than at higher ones (refer to steam table). It is, of course, impossible to give exact figures as they vary with so many circumstances, but generally from 80 to 90 per cent. of the heat supplied to a cylinder in steam goes away in the exhaust steam and about, say, 5 per cent. in heat losses and generally less than 10 per cent. is converted into mechanical energy.

From the principle of the conservation of energy we can write, say, per pound of steam supplied to an engine,

$$\left.\begin{array}{c}\text{Heat supplied to}\\ \text{engine}\end{array}\right\}=\left\{\begin{array}{l}\text{heat equivalent to work done}\\ \quad+\text{heat in exhaust}+\text{heat lost}\end{array}\right.$$

And in a laboratory we can measure all these quantities except the heat lost (e.g. to the atmosphere and in steam leakage) and we can estimate this. If we supply dry saturated steam at a known temperature, then from the steam tables we can write the above equation per pound of steam,

$$\mathbf{H_1 = h_1 + L_1 = h_2 + x_2 L_2 + \text{heat equivalent to work done} \\ + \text{heat lost,}}$$

where x_2 is the dryness fraction of the exhaust steam, h_2 the sensible heat of water, and L_2 the latent heat of steam, both at the exhaust temperature, and H_1 the total heat of dry saturated steam at admission to the cylinder.

In an internal combustion engine commonly from 30 to 40 per cent. of the heat energy in the fuel can be converted into mechanical work. About 30 to 40 per cent. is commonly lost in the hot exhaust gases, and 30 to 40 per cent. in the cooling water employed to keep the metal of the cylinder sufficiently cool to permit of lubrication (or in air if air cooling is employed as in aviation).

So far as the *proportion* of heat converted into work is concerned the reciprocating steam engine, particularly in small sizes, does not compare well with the internal combustion engine but there are many other considerations than this which go to determine the choice of a power plant, notably the cost of fuel in a particular locality. And circumstances still exist which give the relatively small steam plant an opportunity to be the most economical. A large proportion of the heat supplied to a steam engine is finally discharged in the cooling water employed to condense the steam in the condenser. But if, for an industrial purpose power is required and also a large supply of hot water, then the exhaust steam, a by-product of the power or mechanical energy production, can be employed to heat water which is *not* wasted and the use of fuel in this way for two purposes may prove more economical than any other method of meeting both requirements.

Example 1. 1 lb. of dry saturated steam at a pressure of 100 lb. per sq. in. is expanded adiabatically to a pressure of 15 lb. per sq. in., and does 10,810 ft.-lb. of work. Find the resulting dryness fraction of the steam after expansion.

Heat before expansion (from steam table) at 100 lb./sq. in. $= 1191$ B.Th.U.

Heat equivalent to work done $= \dfrac{10{,}810 \text{ ft.-lb.}}{778 \text{ ft.-lb./B.Th.U.}}$

$= 139$ B.Th.U.

Heat remaining in wet steam $=1191-139$
$=1052$ B.Th.U.

Sensible heat in water at 15 lb./sq. in. (from tables) $=181$ B.Th.U.

Latent heat in wet steam after expansion, $1052-181$ $=871$ B.Th.U.

Latent heat in 1 lb. dry steam at 15 lb./sq. in. (from tables) $=970$ B.Th.U.

Proportion dry or dryness fraction $=\frac{871}{970}=0{\cdot}898$
or 89·8 per cent.

Example 2. If in Example 1 the steam is produced by heating the condensate from the expanded steam, what proportion of the net heat energy supplied is converted into mechanical energy?

Total heat in 1 lb. of steam supplied to cylinder $=1191$ B.Th.U.

Sensible heat in water to be evaporated, per pound $=181$ B.Th.U.

Net heat supplied per pound, $=1191-181$ $=1010$ B.Th.U.

Heat equivalent to work done per pound $=139$ B.Th.U.

Proportion of energy converted $=\frac{139}{1010}$
$=0{\cdot}138$ or 13·8 per cent.

Example 3. Steam at 120 lb. per sq. in. is used in a steam engine cylinder without expansion and the exhaust pressure is 5 lb. per sq. in. Find the work done per pound of steam used, assuming the steam is dry and saturated. If the steam is evaporated from water at the temperature of the exhaust, neglecting effects of condensation, find what proportion of the net heat energy supplied is converted into mechanical energy.

Effective pressure $=120-5=115$ lb. per sq. in.

Increase in volume of 1 lb. of water on evaporation is practically the volume of 1 lb. of dry saturated steam at 120 lb. per sq. in. given by the steam table as 3·727 cu. ft. The work done in evaporation and in non-expansion working in the cylinder is 120 lb. per sq. in. ×144 sq. in. per sq. ft. ×3·727 cu. ft. =64,400 ft.-lb.

Heat equivalent of this work is $\frac{64,400}{778}$ =83 B.Th.U

From the steam tables—

Total heat in 1 lb. of dry saturated steam at 120 lb./sq. in. =1194 B.Th.U.

Sensible heat in exhaust at 5 lb./sq. in./lb. (from tables) = 130 B.Th.U.

Net heat supplied per lb. =1194 −130 =1064 B.Th.U.

Proportion of energy converted=$\frac{83}{1064}$=0·078
=7·8 per cent.

Summary of Chapter VI

Adiabatic Expansion and Compression. For a given increase of volume, the pressure of a gas cooled by adiabatic expansion falls more than if its temperature were maintained constant; hence the p-V adiabatic curve is steeper than the isothermal. In adiabatic compression the molecules of a gas rebound from an approaching boundary, such as a piston, with increased speed and kinetic energy and the gas is heated.

Principle of the Internal Combustion Engine. Addition of Energy to Compressed Air. If air is compressed adiabatically, its temperature rises. If it could then be adiabatically expanded the energy given to it in compression could all be recovered. But the energy in compressed air can be diminished by cooling or increased by heating, at constant volume. If it is cooled and then expanded adiabatically it may thereby be cooled below freezing point. But if it is heated and then adiabatically expanded, not only the energy put into it by compression but also some of the energy imparted to it by heat may be recovered as mechanical energy. This is the principle of the air engine and of the internal combustion engine, the working substance of which is largely air.

Compression and Expansion of Vapours. The effect of compression or expansion on saturated or wet vapours differs from that on air in that it increases or decreases the mass of vapour by evaporating or condensing some of it by heating or cooling.

Work done in Evaporation. In addition to the greatly increased kinetic energy of the molecules when water is evaporated to steam, work (equal to $p \times V$) is done in making room for the increased volume occupied by the steam. This work may be done in pushing a piston in a cylinder and is part of the operation of a steam engine.

Principle of the Steam Engine. The steam engine employs the work done in vaporisation of steam; it also extracts some of the energy from the steam by expansion when the flying molecules give up kinetic energy by impinging on a receding piston and rebounding with diminished speed, and it may employ condensation to reduce the atmospheric resistance to the motion of the piston. The ideal operation may be illustrated by a p-V diagram. The actual conditions differ from the ideal in various ways, some of which can be seen from the indicator diagram, which also is a measure of the work done by the steam.

While the latent heat of steam enables it to carry much energy without exceedingly high temperature, most of the heat passes away in the low pressure steam which goes to the exhaust and the efficiency is not high.

EXAMPLES VI

1. Find the work done in forming 1 lb. of dry saturated steam at 100 lb. per sq. in. pressure and hence find in B.Th.U. its gain in internal energy. (Refer to steam table for specific volume and latent heat.)

2. The rated horse-power for taxation purposes of a 4-stroke petrol engine is, per cylinder, horse-power $=0{\cdot}4$ (cylinder diameter in inches)^2. If the piston speed is assumed to be

1000 ft. per min., what is the assumed m.e.p. in pounds per square inch?

3. A 6-cylinder, 4-stroke cycle, petrol engine has cylinders 61·5 mm. diameter, 100 mm. stroke. What brake horse-power will it develop at 1800 r.p.m. if the indicated m.e.p. is 135 lb. per sq. in. and the mechanical efficiency is 90 per cent.?

4. A 4-stroke cycle motor car engine has 4 cylinders each 2·5 in. diameter and 3·74 in. stroke. If it can develop 22 b.h.p. at 2000 r.p.m. find the brake m.e.p. What would be the horse-power rating for taxation purposes? (See No. 2.) If under ideal road conditions this engine only needs to develop 5·25 b.h.p. at 30 m.p.h. and uses petrol at a rate of 54 miles per gal., find the consumption in pints per brake horse-power-hour and in pounds per brake horse-power-hour and the brake thermal efficiency, if the specific gravity of the petrol is 0·74 and its calorific value is 19,500 B.Th.U. per lb.

5. A 7-cylinder radial petrol aero engine (4-stroke cycle) runs at 2500 r.p.m. and develops 154 b.h.p. The cylinders are 4·25 in. diameter and 5 in. stroke. Find the indicated m.e.p. on the assumption that the mechanical efficiency is 88 per cent.

6. Find the diameter of cylinder and length of stroke for a double-acting steam engine to develop 35 i.h.p. with a piston speed of 450 ft. per min., the initial gauge pressure being 90 lb. per sq. in., cut off at $\frac{1}{4}$ stroke back pressure 4 lb. per sq. in. absolute and a diagram factor of 0·75. The stroke to be 1·5 times the cylinder diameter approximately.

7. 1 lb. of dry saturated steam at 100 lb. per sq. in. pressure is adiabatically expanded till its pressure has fallen to 15 lb. per sq. in., when its dryness fraction has become 90 per cent. How many foot-pounds of work has been done by the steam in its expansion?

CHAPTER VII

ENGINES

66. Steam Engine History and Types. The steam engine followed the atmospheric engine which, with very large vertical cylinders, utilised the pressure of the atmosphere in the upper side of a piston when the pressure on the lower side was reduced below that of the atmosphere by first filling the cylinder with steam and then condensing it with cold water. The process was slow and wasteful of heat and the means of making cylinders and pistons to fit them were very crude in the eighteenth century, when such engines were used. Most notable in the development of the steam engine was James Watt, an instrument maker who contributed many ideas and inventions, and in particular that of the use of a separate condenser designed, like many subsequent improvements, to reduce the loss by condensation of the steam before it has had its heat converted into mechanical energy. Like the atmospheric engines, the early steam engines were used for pumping water out of mines and the development for this and other purposes is a fascinating and instructive story. For about a century the steam engine and boiler held the field almost unchallenged, as the means of obtaining mechanical energy from the combustion of fuel; and during this period it took the form only of the reciprocating engine, that is, one in which a piston moved to and fro in a cylinder and the rotatory motion, if required, was produced from the linear motion by mechanism, generally of the well-known connecting rod and crank type. During the second half of the nineteenth century the rotatory or turbine type of engine began to be developed and has become the outstanding type of large steam engine, the study of which must be left to a later stage. Meanwhile, mainly also in the second half of the nineteenth century, the internal combustion engine was being developed and has now largely displaced the steam engine in certain fields of use, particularly for small and medium powers. For rail locomotion the reciprocating steam engine and boiler are still largely used and also for the propulsion of many moderate-sized merchant vessels, particularly of the tramp class. For the production of power on a large scale as in electrical power generating stations, the steam turbine is widely used; it shows a good steam economy in large but not in small sizes.

Reciprocating steam engines can be classified in many ways. Thus they can be put into two classes as condensing and non-condensing. Nearly all are double-acting but single-acting steam engines are not unknown. Engines are also classed according to the type of valves employed, which varies with the size and speed of running. At this stage it must suffice to consider a simple single cylinder engine of relatively small size because it is not like, say, the locomotive engine, specialised, and its form determined by special circumstances of position and purpose. But it is just this small general purpose engine which has been largely replaced by one form or another of internal combustion engine or by electric motors.

67. Single-Cylinder, Double-Acting Steam Engine. While this article may suffice to explain how one simple type of engine works, it is very desirable that a sectional model should be examined in a laboratory to see just how the valve controls the steam inlet to, and the exhaust from, the cylinder.

The operation of the steam within the cylinder has been explained in Art. 63 and Figs. 52, 53, 54, 55, and 56, but the operation of one type of valve and its control of the steam admission, expansion, and exit is illustrated in Fig. 57. This shows diagrammatically only a single-cylinder double-acting steam engine with a slide valve. The slide valve has been used for a long period and still is used on small stationary reciprocating steam engines and on locomotive engines. On one side of the engine cylinder is cast a chamber, called the steam chest, to which the steam is led by pipes from the boiler. Steam passages lead from the steam chest to each end of the cylinder. The entrances to these passages from the steam chest are through rectangular openings or steam ports, narrow as seen in the sectional diagram, but long in a direction perpendicular to it. These steam ports are opened and closed by the reciprocating movements of the slide valve which is driven from the crankshaft by an eccentric rod actuated by an eccentric, which is in effect a crank of small throw. The full travel or stroke of the valve is twice the throw of the eccentric. Not only does the slide valve by its outside edges control the admission of steam to the cylinder when the piston is at or near the end of its stroke, but it also (when moving in the opposite direction, on its return stroke) cuts off the steam supply at a pre-arranged fraction of the piston's

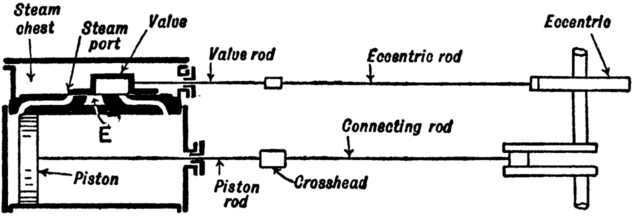
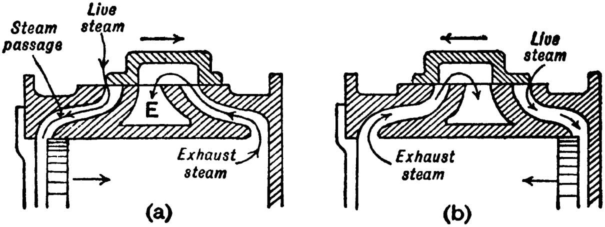


FIG. 57.—Diagram of double-acting engine with slide valve.



(a) (b)

FIG. 58.—Section of slide valve, ports and passages.

motion. Furthermore by its inside edges it opens and closes the ports and allows the used steam to escape to the exhaust passage E. In passing from the cylinder to the exhaust, the steam returns along the curved steam passages, enters the hollow box-like part of the slide valve and thence passes through the relatively broad exhaust port to the exhaust passage E. Enlarged diagrams of the valve, ports, and passages are shown at (*a*) and (*b*) of Fig. 58 for both ends of the piston's stroke.

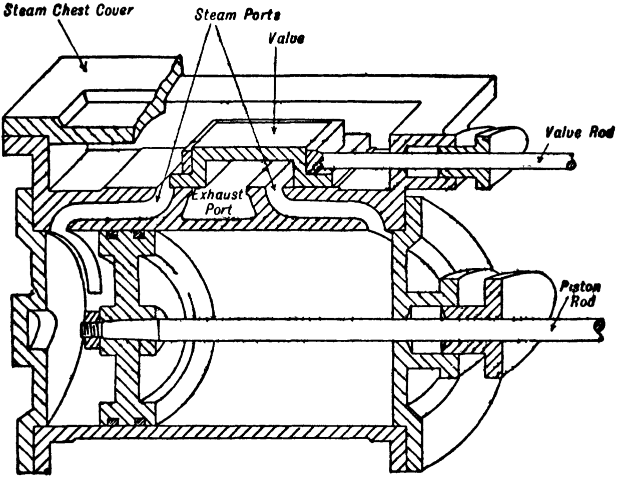


FIG. 59.—Pictorial section of slide valve, steam chest and ports.

A pictorial sectional diagram is shown in Fig. 59. In this the piston has started its outward stroke and the steam port is nearly fully open while the passage at the other end of the cylinder is fully open to exhaust.

68. Engine Auxilliaries. (1) *Condensers.* A non-condensing engine allows the released steam to flow into the atmosphere after expansion when the valve opens the exhaust port. As the passages and ports are of limited size, the back pressure is somewhat above that of the atmosphere and this pressure forms a resistance to the motion of the piston or a reduction in the effective force urging the piston forward. In a condensing engine a vacuum, or pressure below atmosphere, is produced in the cylinder after the opening of the exhaust port. This is done by condensing the steam, not in the cylinder itself, but in a separate condenser to which the steam rushes on release. Condensers are of two main kinds. The older type is the jet condenser, in which the exhaust steam comes into actual contact with jets of cold water. This is illustrated diagrammatically in Fig. 60. After condensation it is necessary to remove the water (including the condensed steam) by means of a pump. This pump has to be of much larger capacity than would be necessary to remove the injected water and condensed steam only. It has to remove air which finds its way into the engine cylinder with the steam ; for the water fed into the boiler always contains air dissolved in it and there may also be air leakages into the condenser, which is much below atmospheric pressure. If air were not removed, it would accumulate and quickly spoil the vacuum produced by condensing the steam. Consequently the pump which empties the condenser is called the " air pump." It delivers the water through a valve into a reservoir called the " hot well " from which, if the water is pure enough, the boiler may be fed with warm water, thus saving some heat.

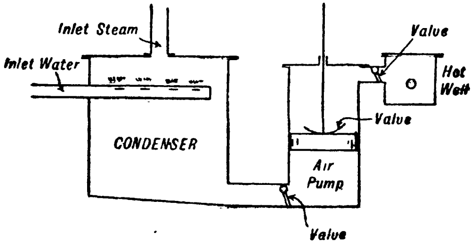


FIG. 60.—Diagram of jet-condenser.

The other type of condenser is the surface condenser. It is shown diagrammatically in Fig. 61. In this, the cooling water and the exhaust steam do not come into actual contact, but the steam condenses on the outer surface of a large number of thin metal tubes conveying the cooling water circulated as shown, by means of a pump called the circulating pump. The track of the cooling water and of the steam through the condenser is indicated by arrows in Fig. 61. A surface condenser is used when it is important not to contaminate the condensed steam with impure water so that it can be returned to the boiler in cases where there may be an ample supply of impure cooling water but a scarcity of

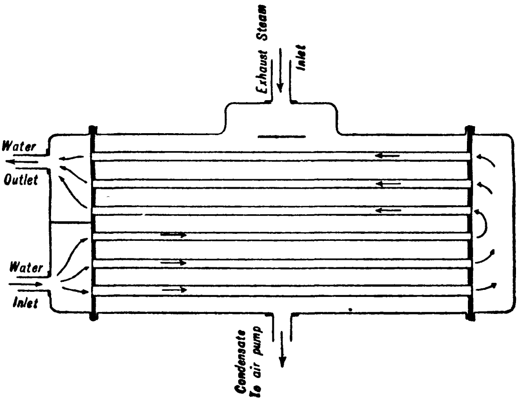


FIG. 61.—Diagram of surface condenser.

pure water for the boiler. A notable case is at sea, where sea water may be used for cooling purposes but would be highly unsuitable for use in the boiler as it would cause corrosion and deposit salts or scale on the inside of the boiler.

With the use of a surface condenser the function of the air pump is to remove the air and condensed steam; the cooling water is pumped in, forced through the tubes and out to waste by means of the circulating pump. Where water is scarce, the cooling water after use is again cooled by the atmosphere in some form of cooling tower in which the air cools the sprayed warm water.

Example 1. A surface condenser receives 1000 lb. of steam per hour at a pressure of 4 lb. absolute and 0·90 dry. If the cooling water is supplied at 60° F. and is not to be above 94° F. on leaving, find how many pounds of cooling water will be required per hour if the temperature of the condensate is 106° F.

The total heat lost by the steam must be equal to the heat gained by the cooling water if there is no loss of heat.

From the steam tables, at 4 lb. per sq. in.—

Latent heat of steam is 1004·9 B.Th.U. per lb.

Latent heat per pound of wet steam is $0{\cdot}9 \times 1004{\cdot}9$ = 904·4 B.Th.U.

Sensible heat per pound of steam = 120·8 B.Th.U.

Total heat per pound of wet steam = 1025·2 B.Th.U.

Sensible heat in condensate per pound at 106° approximately $106 - 32$ = 74 B.Th.U.

Loss of heat by steam per pound $= 1025{\cdot}2 - 74$ = 951·2 B.Th.U.

Heat exchanged per hour $= 951{\cdot}2 \times 1000$ lb. = 951,200 B.Th.U.

Again taking the specific heat of water as unity—

Gain of heat per pound of cooling water = 1 lb. × (94 − 60)° F. = 34 B.Th.U.

Weight of cooling water required

$$\text{per hour} = \frac{951{,}200 \text{ B.Th.U.}}{34 \text{ B.Th.U./lb.}} = 27{,}980 \text{ lb.}$$

Example 2. Steam enters a jet condenser at a pressure of 2 lb. absolute and 0·95 dryness fraction. If the condensing water is supplied at 65° F., find how many pounds of water are required per pound of wet steam if the temperature of the hot well is to be 95° F.

Latent heat of dry steam at 2 lb./sq. in. (from tables) =1019·7 B.Th.U.

Latent heat in 1 lb. of wet steam $=0{\cdot}95\times1019{\cdot}7$ = 968·8 B.Th.U.

Sensible heat per pound at 2 lb./sq. in. = 93·9

Total heat per pound of wet steam =1062·7 B.Th.U.

Sensible heat per pound in condensed and cooled steam at 95° F., $=1\text{ lb.}\times(95-32)^\circ$ F. = 63 B.Th.U.

Loss of heat per pound of steam = 999·7 B.Th.U.

Gain of heat per pound of water, $=1\text{ lb.}\times(95-65)^\circ$ F. = 30 B.Th.U.

Weight of water required per pound of steam $=\dfrac{999{\cdot}7}{30}$ = 33·32 lb.

Example 3. If only 24 lb. of water per lb. of steam is supplied for the jet condenser in Example 2, find the temperature of the hot well.

Let t° F. be the unknown temperature of the hot well.

Sensible heat per pound in condensed steam $=t-32$ B.Th.U.

Loss of heat per pound of steam $=1062{\cdot}7-(t-32)$ $=1094{\cdot}7-t$ B.Th.U.

Gain of heat per pound by cooling water $=t-65$ B.Th.U.

Total gain by cooling water per pound of steam condensed $=24\,(t-65)$ B.Th.U.
$=\cdot 24t-1560$ B.Th.U.

and since this is equal to the loss per pound by the steam

$$24t-1560=1094{\cdot}7-t$$
$$25t=2654{\cdot}7$$
$$t=106{\cdot}2^\circ\text{ F.}$$

(2) *Governors.* If an engine is doing work against a resistance and this resistance or load is removed or greatly diminished, the effort exerted by the steam would be excessive, and unless there were some means of controlling the speed the engine would be accelerated and reach a dangerous speed. The governor has two functions: it is a safety device to prevent excessive speed on sudden removal of load and it is also a means of keeping the speeds for different loads within a certain margin of variation. We shall not describe governors in detail but generally they act by employing the variation, with the speed, of centrifugal force exerted by a metal ball revolving in a circular path, upon hinged links. A rise in speed causes the ball to exert an increased centrifugal force and take up a new running position, thereby operating a lever which operates a valve and curtails the steam supply to the engine cylinder. This is appropriate to the lighter load, the reduction to which was the cause of the increase in speed. Thus there are different speeds corresponding to different loads and unless there be a change in speed the governor does not operate. But the governor is designed to reduce the differences in speed to small proportions for wide variations in the load.

(3) *The Flywheel.* The flywheel also controls the variations of speed of an engine, but whereas the governor limits the changes in average angular speed over such periods as, say, a minute, due to changes in load, the flywheel limits the fluctuations in angular velocity within, say, a single revolution. If there were no flywheel, an engine—particularly a single-cylinder engine—would run jerkily. For the turning moment exerted on the crankshaft is variable due to the change in steam pressure during a stroke and still more to the variation in leverage of thrust of the piston (through the connecting rod) on the crank. The variation in torque on the rotating crankshaft represents a wide variation in the power or rate of output of energy in different parts of a single revolution and unless there is a considerable amount of rotatory inertia, that is, a flywheel, there would be wide variation in speed during

the course of a revolution or even of half a revolution. Put in terms of energy, the flywheel (see Art. 33) is a reservoir of kinetic energy into which the engine delivers energy when it is producing in excess of the average rate, and from which it draws energy when it is producing below the average rate. Provided the reservoir is large enough, the alternate additions and subtractions of energy leave the reservoir of energy near to a constant value, that is, the fluctuations of speed of the flywheel (and with it the engine) are small. The flywheel is further explained with numerical illustration in Art. 75 in connection with the internal combustion engine, which often needs more flywheel control than the steam engine. The need for speed control by means of a flywheel is of course less with more cylinders than with one, because the variation in torque on the shaft can be greatly reduced by arranging that one piston is exerting more than average torque while another is exerting less than average or even zero torque.

69. Simple Steam Engine Plant. The steam engine differs from the internal combustion engine, and particularly from the oil and petrol engines, in needing at least one relatively large and costly additional piece of plant, namely, a steam boiler in which the steam is produced. Consideration of boilers is deferred to the following chapter, but we may here conveniently review the minimum items of a steam engine plant. These are shown diagrammatically in relation to one another in Fig. 62. Following the circuit of the water we may begin at the hot well where the condensed steam from the condenser is delivered by the air pump. Here any additional water from an outside supply will be added to make up for waste by escape of steam or water. And if the engine is non-condensing, the hot well will be replaced by a cold water cistern. The water will first be drawn into the boiler feed pump, driven by steam from the boiler or by outside power, generally in this case electrical. Next the water passes along the feed pipe into the boiler, a part of the plant requiring skilled attention. From the boiler the working

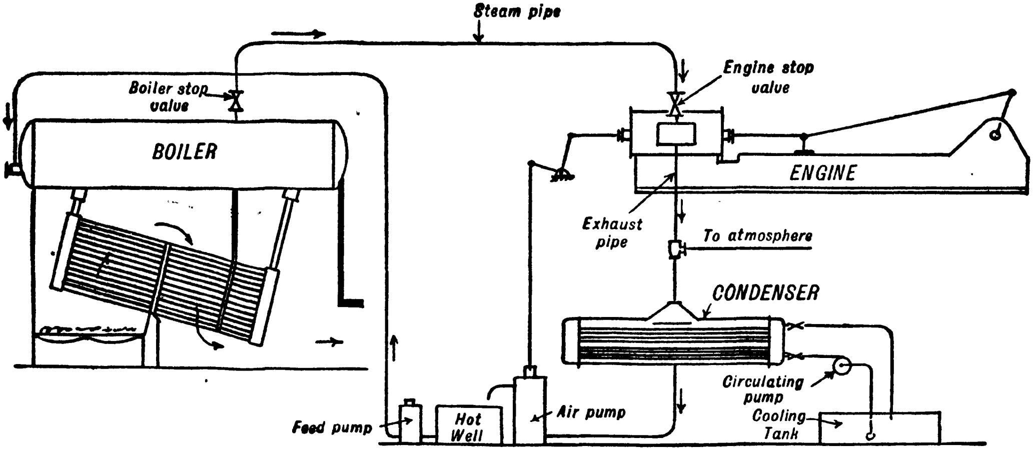


FIG. 62.—Diagram of simple steam engine plant.

substance, now in the form of steam, passes through the steam pipe to the engine, there being hand-operated stop valves to control the supply at exit from the boiler and at entry to the engine cylinder. The steam enters the engine cylinder, does work, and passes to the exhaust pipe. If the engine is non-condensing the steam flows out to the atmosphere, but if it is a condensing engine the steam goes to the condenser. A change-over valve is shown in Fig. 62, by which the exhaust can go direct to atmosphere or to the condenser, as desired. If it goes to the surface condenser, the resulting water or condensate is removed by the air pump and enters the hot well, thus completing the circuit. One other item not in the steam circuit, but necessary, is the circulating pump, which forces water from the cooling tank or other supply through the condenser in an independent water circuit leading back to the cooling tank.

70. Classification of Internal Combustion Engines. We may recall that in internal combustion engines (Art. 59) the working fluid is essentially air and largely the constituent nitrogen, and that the fuel, which by its combustion supplies the heat energy to the compressed air, is burnt in the cylinder. While there is great variety in types of internal combustion engines, we can perhaps best put them in two simple classes, viz.

(*a*) Low compression or spark ignition;
(*b*) High compression, including compression-ignition.

(*a*) In the low compression class, the air and gas or vapour are drawn into the engine cylinder by suction of the piston on its outward stroke and then compressed on the return stroke before ignition of the mixture. Both the economy and the power obtainable from a given size of cylinder are limited by the fact that compression can only be to such a pressure as will not involve heating the mixture (by compression) sufficiently to ignite it. This often limits the compression to about one-fifth or one-sixth of the original volume at atmospheric pressure, but the degree of allowable compression without risk of ignition varies somewhat with the character of the fuel, e.g. rather higher compression can be used with petrol containing a " dope " than without one. A ratio of between one-sixth and one-seventh is used in modern motor cars. This class includes (1) the gas engine, which was at one time the predominant type of internal combustion engine, and (2) the petrol and other spirit engines, in which volatile liquid fuel is broken into a spray by a suction draft created in a carburettor, and then evaporated by the air charge with which it mingles before entering the engine cylinder. It also includes paraffin engines, in which the fuel has to be assisted to evaporate by heat, generally supplied by the exhaust gases. In this class the mixtures of air and fuel gas or vapour after suitably limited compression are ignited by an electric spark and burn, giving out their heat at approximately constant volume.

(*b*) In the high compression class, a charge of air is compressed to a high pressure and thereby also raised to a high temperature, so that when the oil fuel is injected into the highly compressed and heated air, it immediately burns without the need of any external means of ignition. This type is known as the *compression-ignition* engine. It is also called the Diesel type after the name of the German engineer, Rudolph Diesel (1858–1913), who was a pioneer in its development. There is also a type of internal combustion engine using rather lower compression pressures, in which ignition is assisted by a " hot bulb," i.e. a portion of the cylinder end cover which is not water-cooled but is maintained at a red heat, firstly by external heating and later by the heat of the fuel burning in the cylinder. This is called the hot bulb surface-ignition or the semi-Diesel type. It may be regarded as intermediate between the two classes, but belongs to the high rather than to the low compression class. It is associated with the name of Akroyd Stuart, who about 1890 evolved an engine of this type.

Apart from the degree of compression, there is wide variety in internal combustion engines and, particularly, they are divisible according to the cycle of operations which occur in the cylinders, but this is mentioned more fully in Arts. 71 and 74.

Internal combustion engines, except some of large size, are single-acting, that is, the pressure arising from the combustion acts on one side of the piston only, driving the piston on an outward stroke. The piston (Fig. 63) is relatively long compared to that of a double-acting steam engine (in which the piston more nearly approaches a disc form) and, with its gudgeon pin, it fulfils the function of a cross head or slipper to which the open-ended cylinder acts as guide and takes the pressure perpendicular to the direction of motion, arising from the oblique thrust T of the connecting rod (Fig. 63). This arrangement of a cylinder open at one end, for small engines, gives sufficient air cooling of the piston, which is in contact with the very hot burning mixture, to allow of its lubrication. It has also the advantages of simplicity of construction and compactness for high-speed running. Fig. 63, which is diagrammatic only, shows the forces which are exerted on the piston, the upward force R being the reaction of the cylinder to the lateral pressure of the piston on the cylinder.

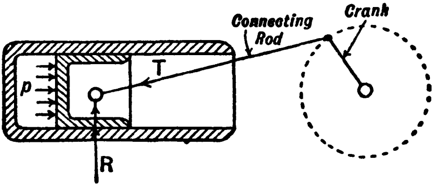
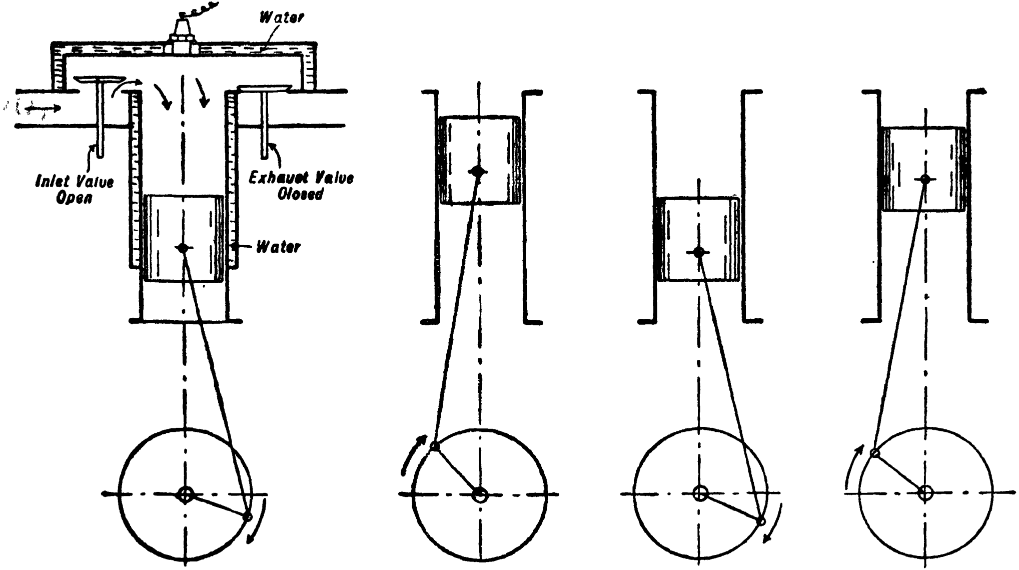


FIG. 63.—Forces on piston of single-acting internal combustion engine.

71. The Four-Stroke Cycle (Petrol Engine). This cycle or series of operations in the cylinder is the commonest one in petrol and gas engines, and is also very widely used in compression-ignition engines. The cycle of events occupies a period of two revolutions of the engine crankshaft and four strokes of the piston. It will be readily understood from the diagrammatic representation shown in Fig. 64. This is purely diagrammatic and the inlet and exhaust or outlet valves



(a) Suction
Inlet open Exhaust closed

(b) Compression
Both Valves closed

(c) Working
Both Valves closed

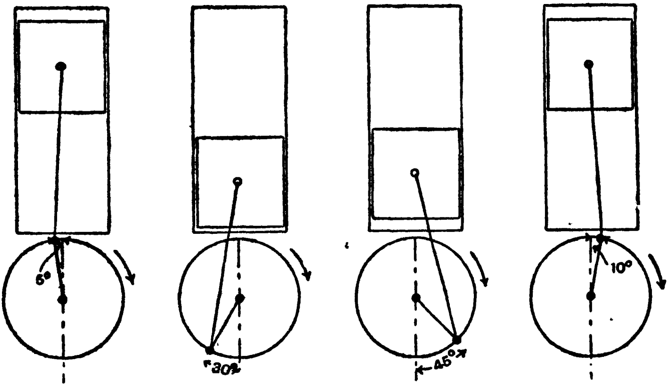
(d) Exhaust
Inlet closed Exhaust open

FIG. 64.—Diagram of four-stroke cycle.

are shown for clearness on opposite sides of the cylinder axis although in practice they are often close together on one side of the cylinder or at the closed end. Diagram (*a*) shows the suction stroke. The piston is descending and creating behind it a suction so that, the inlet valve being open, the inflammable mixture of air and gas or air and vapour from a carburettor is drawn into the cylinder above the piston, completely filling it at the end of this suction stroke. The inlet valve then or shortly afterwards closes and as the piston ascends (diagram (*b*)), both inlet and exhaust valves being closed, the mixture is compressed into, say, one-fifth or one-sixth of the volume which it occupied at the beginning of this compression stroke. About the end of this second (compression) stroke the compressed mixture is ignited by a spark across the points of the sparking plug. The pressure and temperature rise almost instantaneously. The molecules, brought closer to one another (and with somewhat increased velocity) in the compression stroke, have their velocity greatly increased by the combustion, and with their increased speed deliver much greater blows on the piston and cylinder, that is, they exert a greatly increased pressure, and so begins the downward working stroke (*c*) in which the whole work of the cycle is done by the expanding gases as their pressure falls. Near the end of this working stroke the exhaust valve opens and the pressure falls sharply to about that of the atmosphere. The exhaust valve remains open and the piston completes its cycle of operations by the fourth stroke (*d*) upward, expelling nearly all the remaining gases, which include the products of combustion, any excess oxygen over that required in the combustion and, still predominant in quantity, the nitrogen of the air inhaled in the suction stroke. The nitrogen has taken little or no part in the chemical action of the combustion of the fuel, but has taken its share in absorbing released energy as an increase of the kinetic energy of its molecules and in pushing forward the piston in the working stroke. About the end of this fourth or exhaust stroke (*d*) the exhaust valve closes, the inlet valve opens the cycle is complete and another downward, suction stroke, as at (*a*), begins. Let us summarise the events:

(*a*) Suction or inhalation stroke.
(*b*) Compression stroke.
(*c*) Firing or working stroke.
(*d*) Exhaust stroke.

Valve Timing. Because valves take time to open, and because incoming and outgoing currents of air and gases possess inertia, the precise times of valve openings and closures are not what might be expected from a slow-running engine or a model, and vary somewhat according to the speed of the engine. Fig. 65 gives some idea of the crank and piston position for the inlet and exhaust valve movements, but these vary with different makes of engine and according to running speeds.



Inlet opens *Inlet closes* *Exhaust opens* *Exhaust closes*

FIG. 65.—Crank positions for opening and closing of valves.

We may show on a crank pin circle, or for clearness on a spiral, the angular position of the crank for all valve openings and closings. This is shown in Fig. 66 and is typical only. The crank position at the instant when ignition of the compressed inflammable mixture takes place may also conveniently be shown on such a diagram. A typical position is shown at I, say 15° before the upper dead centre.

It will be noticed that the inlet opens, say 5° crank angle before the end of the exhaust stroke, while the exhaust valve is still open, but the piston would then be within one-quarter of one per cent. of its stroke from the end of its up stroke or uppermost position. The piston is almost at rest and the object of opening so early is to get the valve sufficiently open by the time active suction begins, when the piston is moving quickly [illegible] its down stroke. To what extent the inlet and exhaust valves can thus be open at the same time without losing unburnt vapour depends upon such circumstances as the speed and the positions of the two valves; it is a matter to be settled by experience. The outrushing stream or column of exhaust gases in the exhaust pipe has inertia, and having acquired momentum under the thrust of the higher pressure gas at the release in the firing stroke, or under the thrust of the piston in the upward stroke, it leaves behind it some degree of vacuum or low pressure which assists the clearance or "scavenging" of the cylinder and the entry of the air or mixture of air and vapour. For this reason the complete closing of the exhaust valve is deferred until after the end of the up stroke, say 10° of crank travel beyond the end of the up stroke. Similarly the inertia of the incoming gases, which acquire momentum under the suction of the

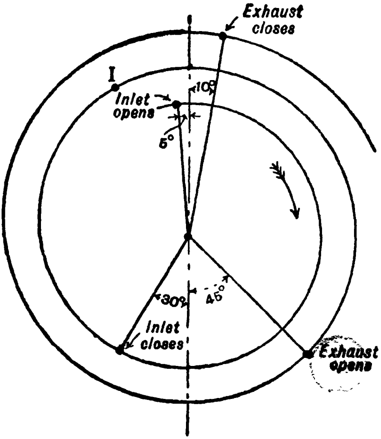


FIG. 66.—Valve timing diagram for 4-stroke petrol engine.

piston, justifies the postponement of the closing of the inlet valve until well beyond the end of the downward suction stroke. It is not desirable that the inlet valve should close until the pressure in the cylinder in the upward compression stroke has risen so far that it would begin to force the mixture out through the inlet valve. This state of affairs is only reached after the piston has begun its upward (compressive) stroke.

Form of the Indicator Diagram. The sequence of operations in the 4-stroke cycle is shown conventionally as a p-V diagram in Fig. 67, in which charging, compression, expansion, and exhaust follow one another in the order shown by the arrows. The pressure during the exhaust is a little above that of the atmosphere and the pressure in the cylinder during suction is of course below atmospheric, thus there is a lower loop on the diagram which represents the network done by the piston, and wasted, in clearing out and refilling the cylinder. This loop is considerably exaggerated in Fig. 67 to make clear the operation of the cycle. On an actual indicator diagram the loop is often too thin to be clearly distinguished from a single pencil line. Otherwise the figure represents the actual indicator diagram of a 4-stroke cycle engine.

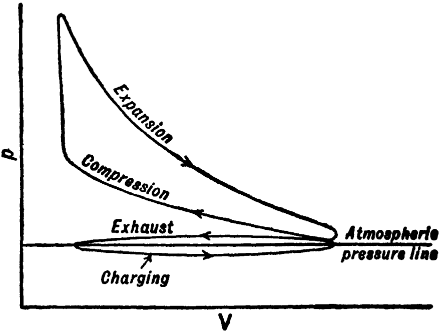


FIG. 67.—Indicator diagram for 4-stroke cycle.

72. The Gas Engine. The preceding article (Art. 71) refers particularly to the petrol or other spirit vapour engine. But with very small differences it applies to engines which employ gas as a fuel. This may be coal gas, natural gas, or gas produced from various organic waste substances or as a by-product in some industrial process.

The only important difference between the petrol and the gas engine is the induction or intake of air and fuel. A gas engine normally has separate gas and air inlet valves so that the explosive mixture is not made until both air and gas are in the engine cylinder or in a chamber connected with it. This affects the valve timing, for the air admission often begins a little earlier than gas admission and if it is so early that a little air passes through the still open exhaust valve, no fuel is lost, and the air current may help to complete the scavenging of the cylinder, that is, to clear out from it of any remaining exhaust gases. Also the air inlet valve may close a little later than the gas inlet valve.

The gas engine was at one time numerically and in respect of total power the predominant form of internal combustion engine, but it no longer occupies that relative position. For small, fast-running internal combustion engines, as in motor vehicles, the petrol engine is most widely used and for large power units the compression-ignition oil engine has become the prevailing form. In special geographical or other circumstances gas engines are still used. In the case of large and relatively slow-running gas engines, the valve timing will be considerably different from that shown in Fig. 66 for the smaller and faster running petrol engine. The valve openings and closings will be nearer to the ends of the stroke.

73. The Compression-Ignition Engine (Diesel). This form of oil engine widely known as the Diesel, after Rudolph Diesel, a German (1858–1913) who took a prominent part in its development, is used in the 4-stroke cycle type, but not exclusively so, for there are important 2-stroke Diesel engines. It differs from the 4-stroke low-compression petrol and gas engines in the suction and compression strokes and in the combustion; the cycle is illustrated diagrammatically in Fig. 68. In the suction strokes, it takes in air only. In the compression stroke the ratio of compression is much higher, say, 14 or 16 to 1 (against 5 or 6 to 1 in the low compression engine) and the air is thereby raised to a pressure of 480 to 500 lb. per sq. in. and to a temperature sufficient to ignite the fuel. This is injected under pressure at the end of the compression stroke in the form of a fine spray from a fuel injection valve, fed by a high-pressure pump. There are two forms of injection. The older form of air blast, in which the oil is blown in by highly compressed air, and the later form called "solid injection" in which the liquid oil is forced into the cylinder through a nozzle

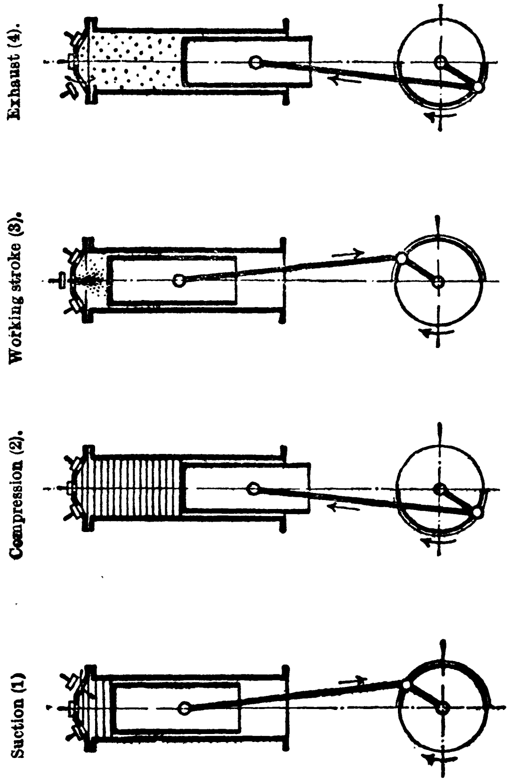


FIG. 68.—Four-stroke cycle of compression-ignition engine.

in spray form without air by a special fuel injection pump which also measures the charge of fuel and can regulate it to requirements by the action of a governor. Combustion is spread over a greater time than in the low compression class of engine. It is not at constant volume, but by spreading the injection over a short portion of the working stroke the combustion is at nearly constant pressure. This is shown in the nearly flat top of the indicator diagram illustrated in Fig. 69. The subsequent events of expansion, release and exhaust do not differ from those in the low-compression class (petrol and gas engines).

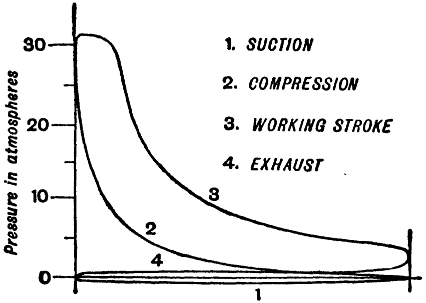


Fig. 69.—Indicator diagram of 4-stroke cycle compression-ignition engine.

The timing of the air inlet and exhaust valves will not vary greatly from that of the corresponding valves of a petrol engine of similar size and speed, e.g. see Fig. 66; and the oil injection may have an average lead or advance of about 10° of crank angle before the end of the upstroke or inner dead centre of the crank pin not greatly different from the point I on Fig. 66. Below full load, the reduced charge controlled by the governor will occupy a smaller time of injection than at full load, and the breadth of the indicator diagram will be reduced accordingly.

Compression-ignition engines are used for high powers, e.g. for electrical power generation and for marine propulsion, and for this purpose some are double-acting engines, which necessitates water cooling of the piston on account of the high temperatures to which it is exposed on both sides. They are also widely used, in the single-acting form, for general purposes in smaller sizes down to those employed for driving the heavier class of motor vehicles.

The compression-ignition engine, because of its high ratio of expansion, attains the high efficiency of 40 per cent. or more reckoned on the ratio of the heat equivalent of the mechanical energy produced, and measured on a brake, to the heat energy in the fuel consumed. In this, it stands in the highest position of any type of reciprocating engine in common use and 25 to 30 per cent. better than the low compression gas or petrol engine. Moreover, it can be made to use a relatively cheap fuel in the form of crude oil and is thus an economical means of producing power in many regions where oil is available. Although it weighs more than a petrol engine of equal power, since it converts a larger proportion of the energy in fuel into mechanical energy, its fuel weighs less. Consequently for a journey above a certain length, the total weight of engine plus fuel is less than for a petrol engine and this gives it an advantage for long-range flight. In addition, the fact that its oil fuel is so much less inflammable than petrol gives it an advantage for aviation.

The hot-bulb or surface ignition engine is similar to the 4-stroke compression-ignition engine, but works with lower compression pressures, say, 200 to 300 lb. per sq. in., and ignition is assisted by the hot surface of a bulb or chamber constituting part of the combustion chamber of the engine cylinder, the temperature of which is maintained by the combustion. The invention of this form of compression ignition engine by H. Akroyd Stuart in England about 1890 in some respects anticipated that of Diesel in Germany, who took out his notable patent in 1892.

74. The 2-Stroke Cycle. This cycle has one working stroke per revolution, that is, twice as many working strokes as the 4-stroke cycle. The working and compression strokes are much as in the 4-stroke cycle, but there are no exhaust or suction strokes. Instead of these, near the end of the working strokes and after release of the exhaust gases, compressed air enters the cylinder and drives out the remaining products of combustion and supplies the cylinder with a fresh charge of air in the case of a compression-ignition engine, or of mixed air and fuel in the case of a low-compression (petrol) engine. Mechanical arrangements of the engine are simplified and in many cases there are no valves, admission and exhaust taking place through ports in the cylinder when these are uncovered by the piston, which is made sufficiently long thus to serve instead of a valve. There are various arrangements for compressing the air charge and admitting it to the cylinder. In large engines, there may be a separate compressing pump and a mechanically operated admission valve. But an arrangement commonly adopted in small engines is illustrated diagrammatically in Fig. 70. In this, the (lower) open end of the piston serves as a pump and compresses the air in a closed crank chamber. When the piston is near the end of its (downward) working stroke, it uncovers the exhaust ports in the lower part of the cylinder leading to the exhaust pipe,

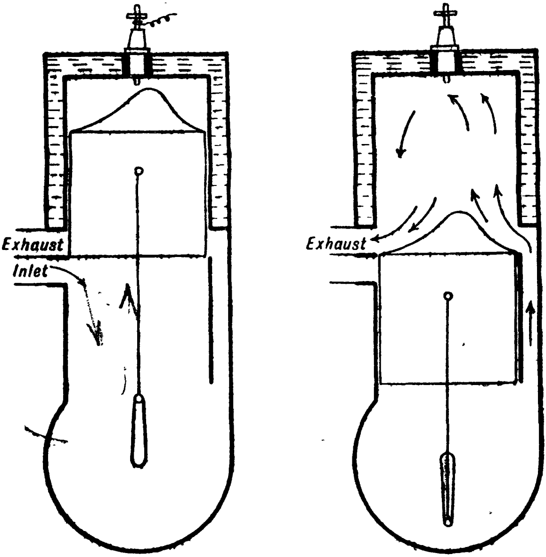


FIG. 70.—Diagram of 2-stroke cycle.

and also uncovers openings on the opposite side of the cylinder communicating with the closed crank chamber, the air or mixture in which has been compressed by the piston in the downward stroke.

The air or mixture flows through the transfer port into the cylinder when it is deflected (upward) towards the far end (top) of the cylinder by a hump or deflecting surface on the end of the piston. It thus sweeps out in front of it the remaining gaseous products of combustion which pass through the exhaust ports, which should be closed (i.e. covered by the piston in its upward stroke) before any incoming unburnt vapour escapes by the same route (if the engine is of the low-compression type using gas or volatile fuel such as petrol). This operation of *scavenging*, or driving out the products of combustion, can be made more complete without risk of loss of fuel if the engine is of the compression-ignition type, for the fluid admitted for compression is then simply air. The 2-stroke cycle for a compression-ignition engine with separate air admission valves is shown diagrammatically in Fig. 71.

It is in connection with the 2-stroke engine that there exists the greatest scope for ingenuity in utilising the inertia of the column of exhaust gases to secure a suction in their wake which shall assist the quick admission of a fresh charge of air or mixture of air and gas.

The advantages of the 2-stroke type of engine are: the much greater power obtainable from a cylinder of given size, the simplification of construction by replacing mechanically operated valves by piston control of cylinder ports, and the fact that it is possible to run in either direction (and to reverse) by adjustment of the ignition. With two working strokes in 2 revolutions as compared with only one in the 4-stroke cycle, there is steadier running, that is, smaller speed fluctuation, for a given size of flywheel or, for the same speed fluctuation, the economy of a smaller flywheel. The disadvantage, with the low-compression type, is the escape of some unburnt fuel down the exhaust. This may be largely obviated for some particular condition of running, say, full load, but will generally involve loss at some other load or speed.

The 2-stroke cycle types of some marine engines are double-acting and thus much power is produced from a given size of engine with two working strokes per revolution. Some very large engines of this type have been built, and the heat in the cooling water is utilised for central heating on land or for raising steam at sea.

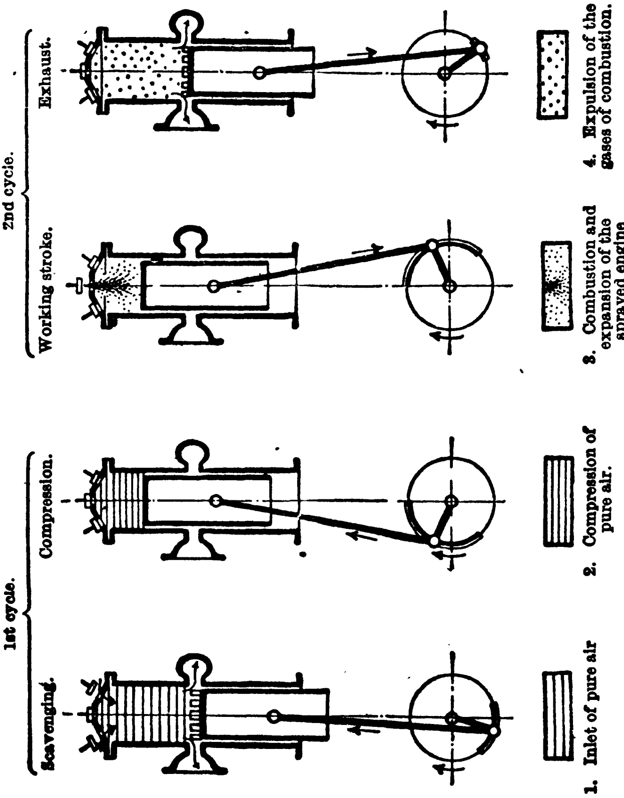

1st cycle.
Scavenging.
Compression.
2nd cycle.
Working stroke.
Exhaust.
1. Inlet of pure air
2. Compression of pure air.
3. Combustion and expansion of the sprayed engine oils.
4. Expulsion of the gases of combustion.

FIG. 71.—Two-stroke cycle for compression-ignition engine.

75. Exchanges of Energy and Fluctuation of Speed. In describing the 4-stroke cycle it was explained that in only one stroke out of four the piston is driven by the high pressure of the hot gases behind it. In the other three strokes, of exhaust, suction, and particularly in compression, the piston moves and does work for which it required energy. This is, of course, supplied by the kinetic energy of the moving parts of the engine and particularly by the flywheel, which is provided to secure steadiness of running or to obviate great fluctuations of speed within a cycle of events. With several cylinders all driving the same crankshaft, the variation in driving effort can be greatly reduced, but for a single cylinder 4-stroke cycle engine the need of a flywheel to control the speed fluctuation within a cycle is exceptionally great. It is particularly easy in such a case to make an approximate estimate of the size of flywheel required to control the cyclical speed fluctuation within given limits.

Consider what happens to the energy transferred from the molecules of the working fluid to the piston in one working stroke. It has to drive the engine and do external work on whatever the engine may be driving, throughout *four* strokes. Hence three-quarters of the energy of one working stroke must be temporarily stored to serve for the three following strokes, while only one quarter is used during the working stroke. In order to increase its store of kinetic energy during the working stroke, the flywheel must be accelerated and will reach its maximum speed towards the end of the working stroke and will slow down to its minimum speed near the beginning of the following working stroke, when it begins to be accelerated again. During the acceleration, driven by the piston, it will be gaining kinetic energy, and during deceleration, or fall of speed, it will be losing kinetic energy which is being used to drive the engine and overcome external resistance. And at all times the kinetic energy of the flywheel will be proportional to the square of its angular speed. As shown in Art. 33, if the speed increases by 1 per cent. the kinetic energy will increase by 2 per cent., for if the engine were running at N revolutions per min., $(1{\cdot}01N)^2 = 1{\cdot}02N^2$ approximately. Suppose the cyclical speed fluctuation is to be kept within 1 per cent., that is, the greatest speed is not to exceed the minimum by more than 1 per cent. Then the maximum kinetic energy must not exceed 1·02 times the minimum, or the fluctuation of energy must not exceed 0·02, or one-fiftieth part, of the minimum kinetic energy during a cycle. We have already seen that this energy, stored during each working stroke and returned in the three following strokes, is about three-quarters of the work done in a working stroke. Hence we may write an equation for the cyclical fluctuation in energy.

Work done in N revs., that is, in N/2 working strokes

$$= \text{horse power} \times 33{,}000 \text{ ft.-lb.}$$

Work done in one working stroke

$$= \frac{\text{horse power} \times 33{,}000}{N/2} \text{ ft.-lb.}$$

Hence $\frac{1}{50}$ of minimum kinetic energy

$$= \tfrac{3}{4} \times \frac{\text{horse power} \times 33{,}000}{N/2} \text{ ft.-lb.}$$

or minimum kinetic energy

$$= \frac{50 \times 0{\cdot}75 \times \text{horse power} \times 33{,}000}{N/2} \text{ ft.-lb.}$$

In this, it is usual to take the horse-power as the i.h.p., which allows for a larger wheel than if we took the b.h.p.

And if W is the weight of the flywheel rim and k its radius of gyration in feet, the effective velocity v of the moving rim is ωk or $\frac{2\pi k N}{60}$ ft. per sec. and its kinetic energy is (Art. 33)

$$\tfrac{1}{2}\frac{W}{g}v^2 = \tfrac{1}{2}\frac{W}{g}\left(\frac{2\pi k N}{60}\right)^2 \text{ ft.-lb.}$$

We may neglect any difference between the minimum speed and the speed denoted by N, which would generally be the average speed, and write, from the two preceding equations—

$$\frac{1}{2}\frac{W}{g}\left(\frac{2\pi N}{60}\right)^2 k^2 = \frac{50 \times 0{\cdot}75 \times \text{horse power} \times 33{,}000}{0{\cdot}5N} \text{ ft.-lb.}$$

We will not reduce this to a formula, but if we assign a suitable diameter for the wheel, the equation will enable us to find a suitable weight W to keep the fluctuation of speed to 1 per cent. For any fluctuation other than 1 per cent. we can multiply the right-hand side by the percentage fluctuation (e.g. by 3 for a 3 per cent. fluctuation from minimum to maximum, which may also be specified as 1·5 per cent. on either side of the mean speed).

It may be noted that if a 4-stroke engine is governed on the "hit and miss" principle of taking in always approximately the same charge, but missing as many of the possible working strokes as may be necessary to prevent the speed rising beyond a given amount, the speed fluctuation will be much greater (unless a larger flywheel is provided). For the change in energy to be given to, and received from, the flywheel will be greater than three-quarters of that per cycle of 2 revolutions. For example, if the governor cuts out alternate working strokes there would be only 1 in 4 revolutions, that is, 8 strokes, and the energy fluctuation between 1 working stroke and 7 others would be seven-eighths of the work per 4 revolutions, or $\frac{7}{8} \times \text{h.p.} \times 33{,}000 \times 4/N$ ft.-lb.

Example. A single-cylinder single-acting I.C. engine working on the 4-stroke cycle develops 15 i.h.p. at 250 r.p.m. Assuming that there is an explosion every cycle, find what weight will be required in a flywheel if the radius of gyration is 2·5 ft., if the fluctuation of speed is not to exceed 2·5 per cent. from minimum to maximum (or 1·25 per cent. from the mean).

$$\text{Work per working stroke} = \frac{15 \times 33{,}000}{125} = 3960 \text{ ft.-lb.}$$

$$\text{Work to be stored} = \tfrac{3}{4} \times 3960 = 2970 \text{ ft.-lb.}$$

With 2·5 per cent. fluctuation of speed there will be 5 per cent. fluctuation of kinetic energy.

Therefore

$$\tfrac{5}{100}\times\text{kinetic energy}=2970\text{ ft.-lb.}$$

Kinetic energy of wheel $=20\times2970$ ft.-lb. $=59{,}400$ ft.-lb.

The linear speed at the radius of gyration is

$$2\pi\times2{\cdot}5\times\tfrac{250}{60}\text{ ft./sec.}$$
$$=65{\cdot}45\text{ ft./sec.}$$

If W = weight in pounds, the kinetic energy

$$\tfrac{1}{2}\,\frac{W}{32{\cdot}2}\times(65{\cdot}45)^2=59{,}400\text{ ft.-lb.}$$

hence $$W=893\text{ lb.}$$

76. Power Measurement. In the earlier days of the steam engine the commonest method of measuring the power developed was the engine indicator (see Art. 64). This interesting instrument, which is still used to some extent to indicate the pressure changes and the working

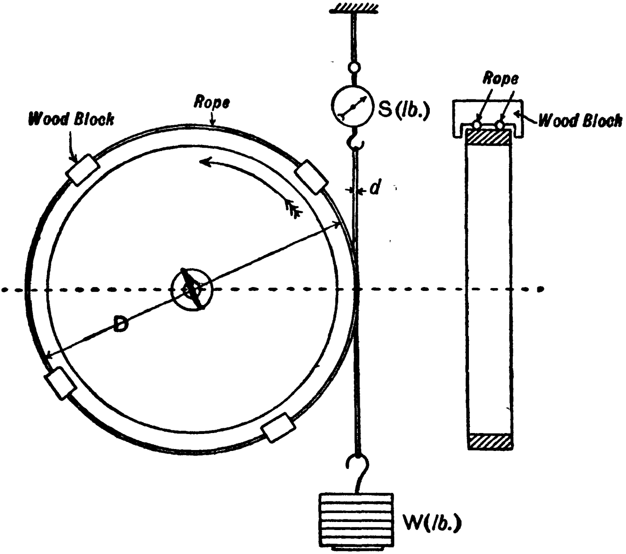


FIG. 72.—Simple rope brake.

of the valves, should be examined and used in the laboratory, but except on large slow-running steam engines, it is rarely used as a power measurer.

It is customary to measure the output power of an engine by measuring the torque applied to the crankshaft when the engine is doing work. This torque is measured either by some mechanism which transmits the power (transmission dynamometer) or by some form of brake which absorbs the power and is sometimes called an absorption dynamometer. A very simple type of brake is illustrated in Fig. 72. It consists of a piece of rope passing round the rim of a flywheel driven by the engine and loaded at one end by a load W lb., as shown. If the wheel were at rest and there were no friction, the pull on the other end of the rope would be equal to W. But the frictional tangential pull of the wheel rim on the rope in the direction of rotation, indicated by an arrow, balances most of the pull of the weight W and leaves only a small amount, S lb. say, which is measured by a spring balance S. Thus the total tangential force exerted counter-clockwise by the wheel on the rope, and clockwise by the rope on the wheel, is (W – S) lb. and acting at a radius, say, $\frac{D}{2}$ ft., from the axis of the shaft. The clockwise moment exerted by the brake on the wheel is $(W-S)\times\frac{D}{2}$ lb.-ft. And if the wheel is not being accelerated, but is turning about its axis with constant angular speed, this must also be the magnitude of the counter-clockwise torque exerted by the crankshaft on the wheel. We therefore know the torque exerted, and if we know the speed of rotation we can find the work done in any given time and hence the power. Let N be the number of revolutions per minute. Then,

$$\begin{aligned}\text{Work per revolution} &= \text{torque in lb.-ft.} \times \text{angle in radians}\\ &= (W-S)D\times\pi \text{ ft.-lb.}\\ \text{Work per minute} &= (W-S)D\times\pi N\\ \text{or}\quad &\ (W-S)\times\text{circumference of wheel}\times N.\end{aligned}$$

The diameter D is to be measured to the centre line of the rope; i.e. $D+d$ in Fig. 72. The simplest plan is to measure the circumference by means of a piece of rope of the same diameter d as that used in the brake.

Brakes take various forms and their use should be demonstrated in a laboratory. Fig. 72 is merely to illustrate the principle of a simple type. Blocks of wood are generally employed to keep the rope or ropes in place on the wheel rim. The energy used in overcoming friction mainly goes to heat the wheel rim; and if air cooling is insufficient to prevent an excessive rise of temperature in the metal, water cooling must be employed.

77. Performance and Efficiencies of Heat Engines. The merits of the performance of a heat engine can be assessed in various ways. Looked upon simply as a machine, it is possible to state its mechanical efficiency provided we can measure the work done by the effort and the work done by the machine on its load or resistance. It has already been stated that the indicated horse-power cannot be measured very satisfactorily in many fast-running reciprocating engines. But if we can measure the i.h.p. and the brake horse-power (b.h.p.), these multiplied by 33,000 would give the work done by the steam or gases on the piston per minute and the work done by the engine (say, on a brake) per minute, respectively, as that,

or Brake

$$\text{Mechanical efficiency} = \frac{\text{work done by engine (in 1 min.)}}{\text{work done by steam (in 1 min.)}}$$

or Piston

$$= \frac{\text{b.h.p.}}{\text{i.h.p.}}$$

This fraction is of some importance, but it does not show to what extent the engine is successful in its function of converting heat energy into mechanical energy. If we know the i.p.h., we know how much work is done in a given time by the working fluid on the piston, and we may be able to find how much heat is supplied to the engine to accomplish this work, and this gives a measure of the degree to which it succeeds in its purpose. In an internal combustion engine this is not difficult, for we have only to measure the fuel supplied and its calorific value. The proportion of heat energy in the fuel converted into work, as shown on the indicator diagram, is called the (indicated) thermal efficiency.

Reducing the work per minute to B.Th.U.,

$$\text{Indicated thermal efficiency} = \frac{\text{i.h.p.} \times 33{,}000/778}{\text{lb. of fuel per minute} \times \text{calorific value of fuel in B.Th.U. per lb.}}$$

But since i.h.p. may not be measurable with accuracy, the thermal efficiency may be stated with regard to the b.h.p., the rate of output of useful work, viz.

$$\text{(Brake) thermal efficiency} = \frac{\text{b.h.p.} \times 33{,}000/778}{\text{lb. of fuel per minute} \times \text{calorific value of fuel in B.Th.U. per lb.}}$$

The performance of an engine is often stated in pounds of a particular fuel per b.h.p.-hour and then we can easily state the brake thermal efficiency if we know the calorific value of the fuel. For 1 b.h.p.-hour is equivalent to 33,000 ft.-lb. per min. × 60 min. ÷ 778 ft.-lb. per B.Th.U. = 2545 B.Th.U. Hence,

$$\text{Brake thermal efficiency} = \frac{2545 \text{ B.Th.U.}}{\text{lb. of fuel per b.h.p.-hour} \times \text{calorific value of fuel in B.Th.U. per lb.}}$$

In order to compare engines, their performances are sometimes stated by calculating what is called the brake horse-power mean effective pressure, or briefly brake-m.e.p.. This is $\frac{(\text{b.h.p.}) \times 33{,}000}{l \times A \times N}$ lb. per sq. in., where l is the length of stroke in feet, A is the area of piston in square inches, and N is the number of working strokes per minute. This is rather artificial, as it does not correspond to an actual pressure in the cylinder, but it is a quantity in fairly common use. This arises from the difficulty of obtaining a reliable indicator diagram (and hence indicated m.e.p.) from the cylinders of high-speed engines (see Art. 64).

In the case of the steam engine it is not quite so simple to state the heat supplied to the engine, for though we may measure the weight of steam supplied and its temperature, dryness or superheat, and so know its total heat per pound, in order to know the net supply we have to adopt some convention as to how much is or should be returned to the boiler, and this must be left to a later study of the subject. But if we know the fuel consumption and the whole output of mechanical work for a steam power *plant*, we may state the *overall* thermal efficiency of the plant just as for an internal combustion *engine* above.

Example 1. An oil engine makes 250 r.p.m. and on a flywheel 5 ft. diameter a brake with ropes 1 in. diameter gives a pull of 119 lb. on the tight end and 15 lb. on the slack end. Find the b.h.p.

Net load = 119 − 15 = 104 lb.
Effective radius = 30·5 in. = 2·54 ft.
Torque on shaft = 104 lb. × 2·54 ft. = 264 lb.-ft.
Work per rev. = 2π × 264 lb.-ft. = 1660 ft.-lb.
Work per minute = 1660 ft.-lb. × 250 = 415,000 ft.-lb.

$$\text{Brake horse-power} = \frac{415{,}000 \text{ ft.-lb. per min.}}{33{,}000 \text{ ft.-lb. per min.}} = 12{\cdot}57 \text{ b.h.p.}$$

Example 2. If the engine in Example 1 uses 5 lb. of oil per hour, of calorific value 19,000 B.Th.U. per lb., find the oil consumption per b.h.p.-hour and the brake thermal efficiency. And if the mechanical efficiency be assumed to be 84 per cent., find the indicated thermal efficiency.

$$\text{Oil used per b.h.p.-hour} = \frac{5}{12{\cdot}57} = 0{\cdot}398 \text{ lb.}$$

Heat in this fuel = 19,000 B.Th.U. per lb. × 0·398 lb.
= 7562 B.Th.U.

$$\text{Heat equivalent of 1 h.p.-hour} = \frac{60 \times 33{,}000}{778} = 2545 \text{ B.Th.U.}$$

Brake thermal efficiency = 2545/7562
= 0·337 or 33·7 per cent.

The indicated horse-power will be $12{\cdot}57 \times \frac{100}{84}$ and the indicated thermal efficiency will be 100/84 times the brake thermal efficiency or

$$\tfrac{100}{84} \times 0{\cdot}337 = 0{\cdot}401 \text{ or } 40{\cdot}1 \text{ per cent.}$$

Example 3. If in Examples 1 and 2 the diameter of the cylinder is 8 in. and the stroke 12 in., find the mean effective pressure in the cylinder if the engine works on the 4-stroke cycle.

Indicated h.p. $= \dfrac{12{\cdot}57}{0{\cdot}84} = 14{\cdot}96$, say 15.

Work per minute in cylinder $= 15 \times 33{,}000$
$= 495{,}000$ ft.-lb

Working strokes per minute $= \frac{200}{2} = 100$

Distance travelled per minute by piston effort $= 100 \times 1$ ft. $= 100$ ft.

Mean force on piston in working stroke $= \dfrac{495{,}000 \text{ ft.-lb.}}{100 \text{ ft.}}$
$= 4950$ lb.

Area of piston $= 0{\cdot}7854 \times 8^2$ $= 50{\cdot}3$ sq. in.

Mean pressure per sq. in. of piston $= \dfrac{4950 \text{ lb.}}{50{\cdot}3 \text{ sq. in.}}$
$= 98{\cdot}4$ lb./sq. in.

Summary of Chapter VII

The steam used in the cylinder of a steam engine has its entry, expansion, and exit controlled by one or more valves. The commonest form of control is that of the slide valve, the working of which is illustrated but can be best understood by operating a working model. Condensation of the steam to reduce the back pressure is carried out in either a jet condenser or a surface condenser, from which the air and condensed steam are removed by a pump. The supply of steam to the engine according to its needs is regulated by the governor which serves to prevent wide variations of speed with alterations in the amounts of work which the engine is required to do. The flywheel reduces fluctuations of speed which would otherwise occur during a single cycle of operations such as a single stroke.

Most internal combustion engines employ either (*a*) low compression and spark ignition (of which type the petrol engine is now the commonest representative), or (*b*) high compression which is sufficient without any firing device to ignite the fuel on admission.

Both these classes may be subdivided according as the engines work on the 4-stroke or the 2-stroke cycle. The hot bulb engine lies between the two main classes, the compression being relatively high but not so high as to give ignition reliably without some external heat, which is applied locally to part of the combustion chamber.

Engine power is commonly measured by a friction brake. The frictional torque multiplied by $2\pi N$ gives the work per minute. If also the i.h.p. has been measured by an indicator, the mechanical efficiency can be found from the ratio b.h.p./i.h.p.

EXAMPLES VII

1. A surface condenser receives steam at 2 lb. per sq. in. pressure abs., just dry and saturated and the condensate is to be about 100° F. Find how many pounds of cooling water per pound of steam will be required if it enters at 55° F. and leaves at 84° F.

2. A jet condenser has to condense 1500 lb. of steam per hour, the steam being at 4 lb. per sq. in. abs. and dryness fraction 90 per cent. Find how many pounds of injection water per hour will be required at 60° F. if the allowable temperature of the hot well is 95° F.

3. In an oil engine brake test, the effective diameter of the brake wheel was 3 ft., the load on the rope 110 lb. and on the spring balance at the slack end 6 lb. Find the brake horse-power at 250 r.p.m.

4. If the engine in No. 3 consumes 5·8 lb. of oil per hour, find the brake thermal efficiency, the calorific value of the oil being 19,000 B.Th.U. per lb.

5. If the engine in No. 3 works on the 4-stroke cycle and has a cylinder 8 in. diameter and piston stroke of 12 in., find the mean effective pressure in the cylinder assuming a mechanical efficiency of 85 per cent.

CHAPTER VIII

COMBUSTION AND BOILERS

78. Combustion. Combustion or burning is a chemical action consisting of the union of the substance burnt with oxygen. This process gives out heat, and substances which are burned for the purpose of liberating heat energy by this chemical action are called fuels. The oxygen required for the combustion of fuels is practically always provided by supplying air, of which about 23 per cent. by weight and 21 per cent by volume, is oxygen. The other constituent of air is nitrogen, which forms about 77 per cent. by weight and 79 per cent. by volume, and is chemically inert or inactive at most combustion temperatures and acts merely to dilute the oxygen. It has already been pointed out (Arts. 59 and 71) that in the internal combustion engine cylinder, nitrogen plays its part in propelling the piston after limiting the rise of temperature and pressure resulting from the combustion in which the oxygen and the fuel are the active agents. Similarly nitrogen reduces greatly the temperature which would be produced in a boiler furnace and its dilutent effect is greatly to reduce the temperature of the furnace gases by which the heat is transferred from the fire to the metal of the boiler. The other constituents of the atmosphere at the level of the earth's surface, water vapour and carbon dioxide, are present in very small proportion but neither these nor the traces of other gases are important in connection with any normal form of combustion.

79. Fuels. Fuels may be solid, liquid, or gaseous Coal, coke, and wood are well known solid fuels; oils, and spirits such as petrol and benzol, are common liquid fuels; and town (coal) gas, which is a mixture of combustible gases, is widely used as a fuel.

Most fuels contain as their combustible constituents carbon or hydrogen or both. The hydrogen, in solid and liquid fuels, is commonly combined with carbon as one of the *hydrocarbons*. Gaseous fuels may contain *free* hydrogen, and they frequently contain important proportions of hydrocarbons. Though carbon may exist in fuels as a hydrocarbon, solid fuels, such as coal and coke, consist very largely of solid carbon not in chemical combination with any other element. Coke and charcoal, both of which are important fuels, consist almost entirely of carbon and are made by heating coal and wood respectively in such a way that the hydrocarbons are driven off in a gaseous form without burning the solid carbon. The combustible gaseous products may be collected, purified and used as gaseous fuel, while valuable products present in smaller proportions are extracted in the purification process. Solid fuels always contain one other class of constituent, viz. the incombustible parts which remain as *ash* after the combustible part has burnt off and gone away in gaseous form. Ash is detrimental and its presence in large proportions may render a substance containing carbon and hydrocarbon almost valueless as a fuel. Thus many grades of peat and low grade coals are unsuitable for many purposes, although they contain much combustible material. The earthy matter which forms ash impedes the flow of air and prevents the combustible constituents burning satisfactorily.

Composition of Fuels. Good coals generally contain from 80 to 85 per cent. of carbon, and oil and spirit fuels (including petrol) contain about 85 per cent. of carbon combined with hydrogen. Good coals commonly contain about 5 per cent. of hydrogen (combined with carbon) and oil and spirit fuels from 12 to 15 per cent.

Anthracite, a hard form of coal with little bituminous matter or hydrocarbon, contains about 90 per cent. or more of carbon and only about 3 per cent. of hydrogen. There is no definite dividing line between anthracite and anthracitic coals with high carbon contents. Coals may contain large amount of ash or ash-forming materials, but a good coal for steam raising should not contain more than 4 or 5 per cent.

Gaseous fuels are so variable in their compositions that it would be misleading to quote proportions. The principal constituents are hydrogen, methane (CH_4), and carbon monoxide (CO).

80. Proportions and Products of Combustion. The combination of oxygen with carbon, hydrogen, and hydrocarbons takes place, not in all proportions, but in simple proportions well known from a study of chemistry, and directly related to the atomic weights of the substances concerned. The atomic and molecular weights of the substances with which we are concerned, stated in terms of the weight of an atom of hydrogen as the unit, are as follows :—

	Atomic weight	Molecular weight
Hydrogen (H_2)	1	2
Oxygen (O_2)	16	32
Nitrogen (N_2)	14	28
Carbon (C)	12	—
Water, or steam (H_2O) .	—	18
Carbon monoxide (CO) .	—	28
Carbon dioxide (CO_2) .	—	44
Methane or marsh gas (CH_4)	—	16

Thus a molecule of oxygen consists of 2 atoms and its weight is 2×16 or 32 times as much as an atom of hydrogen; and a molecule of water or water vapour consists of two atoms of hydrogen and one of oxygen and its weight is $2\times1+16=18$ times that of an atom of hydrogen but only 9 times that of the *molecule* of hydrogen, which consists of 2 atoms.

Combustion of Hydrogen. When hydrogen burns in oxygen two atoms unite with one of oxygen, but as the hydrogen is not usually in the atomic condition, but exists as molecules, as does the oxygen, the chemical equation representing the combination to form water vapour is written, $2H_2+O_2=2H_2O$, which stands for 2 molecules of hydrogen (each containing 2 atoms) uniting with 1 molecule of oxygen (containing 2 atoms) to form 2 molecules of water. Writing the weights concerned in this action we have,

$$\begin{aligned} 2H_2+O_2&=2H_2O\\ 2\times2+32&=2\times18\\ \text{or}\qquad 4+32&=36 \end{aligned}$$

and as the *proportions* remain the same however many atoms of hydrogen may be burned, we may divide by 4 to state the proportion by weight as simply as possible and write $1+8=9$, which implies that 1 lb. of hydrogen unites with 8 lb. of oxygen to produce 9 lb. of water. We may note that this union at the rate of 8 lb. of oxygen used and 9 lb. of water (vapour) produced, per pound of hydrogen burned, also necessarily produces a definite amount of heat, approximately 60,930 B.Th.U. or 33,830 lb.-C.H.U. per lb. of hydrogen burnt. In other words, the *calorific value* of hydrogen is 60,930 B.Th.U. per lb. (but of this about one-seventh, representing the latent heat of the steam produced, may be of only limited use for heating purposes).

Combustion of Carbon. Similarly for the complete burning of carbon to carbon dioxide, the equation and proportions are :—

$$\begin{aligned} C+O_2&=CO_2\\ 12+2\times16&=44\\ 1+2{\cdot}67&=3{\cdot}67 \end{aligned}$$

or 1 lb. of carbon unites with 2·67 lb. of carbon to form 3·67 lb. of carbon dioxide, and in doing so it gives out about 14,540 B.Th.U., or 8080 lb.-C.H.U.

There is, however, an important incomplete form of combustion of carbon which may take place in an insufficient supply of oxygen in which the carbon forms carbon monoxide (or the carbon dioxide may, e.g. in passing through a thick fire, unite with hot carbon to form carbon monoxide, but the effect is the same). In this case the equation and proportions are

$$\begin{aligned} 2C+O_2&=2CO \\ 2\times 12+2\times 16&=56 \\ 24+32&=56 \\ 1+1{\cdot}33&=2{\cdot}33 \end{aligned}$$

or 1 lb. of carbon unites with 1·33 lb. of oxygen to form 2·33 lb. of carbon monoxide.

In the union only about 4400 B.Th.U. are given out instead of the 14,540 B.Th.U. in the previously mentioned complete combustion. Thus, in such incomplete combustion, there is a loss of 14,540 − 4400 = 10,140 B.Th.U. (say, 70 per cent.) of the heat. Hence such combustion is wasteful of fuel. Not only so, but the product, carbon monoxide, is an odourless but very poisonous gas, the presence of which in the exhaust gases of motor car engines has been the cause of many deaths in closed garages.

The foregoing combinations with oxygen are all that are of any consequence so far as the production of heat and the provision of air to supply the necessary oxygen are concerned, but there are other chemical actions both in boiler furnaces and in internal combustion engine cylinders which are of importance from other points of view. The presence of sulphur in coal may add slightly to the heat produced in combustion, but it results in gaseous products which are so undesirable as greatly to outweigh any gain from its heating value. And at the high temperature of the internal combustion engine cylinder, nitrogen of the air, and constituents present in the oil or spirit fuels, may enter into the less simple combinations, resulting in the production of corrosive substances (acids). The latter are mainly responsible for wear of the surface of the cylinder, the effect being due to chemical rather than abrasive action.

81. Air required for Combustion. The proportions by weight of oxygen required for burning carbon and hydrogen are given in Art. 80. This oxygen is provided, both in a boiler furnace and in an internal combustion engine, by supplying air to the fuel under conditions of temperature and pressure which make ignition and combustion possible. But the oxygen which actually enters into the chemical action would only be sufficient if none escaped without such intimate contact with the fuel as to ensure combination with it.

In an actual coal fire some air, including oxygen, passes through spaces between pieces of hot fuel without combination and therefore, to get complete combustion, more oxygen, and consequently more air, than the minimum given by the chemical equation, is necessary, and is supplied. This adds to the flue gases and reduces their temperature.

Since air contains about 23 per cent. by weight of oxygen, 1 lb. of air contains 0·23 lb. of oxygen; and to supply 1 lb. of oxygen, 1/0·23 or 4·35 lb. of air will be necessary if no excess were provided. Thus, allowing no excess for oxygen which escapes combination, 1 lb. of hydrogen requires 8 lb. of oxygen for combustion, and will therefore require 8 × 4·35 or 34·8 lb. of air. Similarly, since 1 lb. of carbon requires 2·67 lb. of oxygen for its complete combustion (Art. 80), it will require

$$2{\cdot}67 \text{ lb. of O per lb. of C} \times 4{\cdot}35 \text{ lb. of air per lb. of O}$$
$$= 11{\cdot}6 \text{ lb. of air per lb. of C.}$$

From the foregoing it is easy to compute the minimum weight of air required for burning, say, a pound of fuel containing a known proportion of hydrogen and of carbon, and we might add of sulphur or other combustible elements, but such a refinement has not justification for minute and undesirable constituents of fuels, since the minimum amount of air is in fact greatly exceeded in the actual supply for combustion. We can best show by an example how to calculate the minimum air supply required.

Example 1. A coal contains 87 per cent. of carbon and 4 per cent. of hydrogen, the remainder being incombustible. Find the minimum weight of air necessary to burn one pound of such coal.

The pound of coal contains 0·87 lb. of carbon and 0·04 lb. of hydrogen, hence it will require

$$\begin{aligned}&0{\cdot}87\text{ lb. of C}\times 11{\cdot}6\text{ lb. of air/lb. of carbon}\\&+0{\cdot}04\text{ lb. of H}\times 34{\cdot}8\text{ lb. of air/lb. of hydrogen}\\&=10{\cdot}1\text{ lb.}+1{\cdot}4\text{ lb.}=11{\cdot}5\text{ lb. of air.}\end{aligned}$$

The minimum thus theoretically necessary does not vary widely for different coals, and is about 12 lb. of air per lb. of coal. Actually an excess of 30 to 60 per cent. is allowed, the amount varying with the type of grate and method of firing. Too much air is wasteful for it cools the burned gases more than is necessary, since all the excess air has to be heated by the furnace, and too little air involves loss by incomplete combustion and the formation of carbon monoxide (CO).

Example 2. If the 1 lb. of coal in Example 1 were burnt completely with the minimum amount of air (11·5 lb.), find the proportional composition by weight of the resulting gases, including the residual nitrogen.

Since 1 lb. of carbon with 2·67 lb. of oxygen yields 3·67 lb. of carbon dioxide (CO_2) (Art. 80), the 0·87 lb. of carbon will produce

$$3{\cdot}67\times 0{\cdot}87=3{\cdot}19\text{ lb. of }CO_2$$

and 0·04 lb. of hydrogen will produce

$$0{\cdot}04\times 9=0{\cdot}36\text{ lb. of steam }(H_2O)$$

and the total weight of gases will be

$$11{\cdot}5\text{ lb. (air)}+0{\cdot}87\text{ carbon}+0{\cdot}04\text{ hydrogen}=12{\cdot}41\text{ lb.,}$$

and since all oxygen is used, the gases consist only of nitrogen, CO_2, and H_2O, and the weight of nitrogen must be $12{\cdot}41-3{\cdot}19-0{\cdot}36=8{\cdot}86$ lb.

We can check this result, for the nitrogen, unchanged by the combustion, must be 77 per cent. of the air used or $\frac{77}{100}\times 11{\cdot}5\text{ lb.}=8{\cdot}86$ lb. The gases per pound of coal then are

CO_2	3·19 lb.
H_2O	0·36 lb.
N	8·86 lb.
Total	**12·41 lb.**

And if we divide the weight of each constituent by the total weight we should find the fraction which each represented, e.g. proportion (by weight) of CO_2 is $\frac{3{\cdot}19}{12{\cdot}41} = 0{\cdot}257$ or 25·7 per cent.

Example 3. Suppose 1 lb. of the coal in Example 1 were burnt completely with 18 lb. of air (i.e. more than 50 per cent. above the theoretical minimum). Find the composition of the resulting gases.

The weights of CO_2 and of H_2O will be as before, and the nitrogen will be 77 per cent. of 18 lb., or

N . . .	$= 0{\cdot}77 \times 18 = 13{\cdot}86$ lb.
CO_2	3·19 lb.
H_2O	0·36 lb.
Total of these constituents	17·41 lb.

and the total weight of gases will be

$$18 \text{ lb. air} + 0{\cdot}87 \text{ carbon} + 0{\cdot}04 \text{ hydrogen} = 18{\cdot}91 \text{ lb.}$$

Hence the oxygen in the gases must be

$$18{\cdot}91 - 17{\cdot}41 = 1{\cdot}50 \text{ lb.}$$

which may be checked by subtracting the oxygen which combines with 0·87 lb. carbon, and that with 0·04 lb. of hydrogen from 23 per cent. of the weight of air supplied. By dividing the weight of each constituent of the gases by the total, we can find the proportion or the percentage by weight of each, e.g.

proportion of CO_2 (by weight)

$$= \frac{3{\cdot}19}{18{\cdot}91} = 0{\cdot}1687 \text{ or } 16{\cdot}87 \text{ per cent.}$$

(Note that the percentage by *volume*, which is often experimentally measured, is considerably smaller because CO_2 is heavier than the remainder of the gases, which is largely nitrogen.)

We have seen how a knowledge of the composition of the fuel and the amount of air supplied for combustion enables us to calculate the proportions of the constituents in the resulting or "flue" gases. Similarly a knowledge of the composition of the fuel and of the flue gases enables us to calculate the amount of air supplied. The measurement of the composition of the flue gases is generally made in the proportions by volume, from which the proportions by weight are easily deduced, as we know the relative heaviness or density of the constituents, but we cannot go further into the subject at this stage. But it will be evident that without calculation, a measurement of the percentage of CO_2 present in flue gases will indicate whether too much or too little air is being supplied. As a guide to successful stoking of a boiler fire, automatic recorders of the proportion of CO_2 in the flue gases are used.

Gaseous Fuels. In the case of gaseous fuels, we may be more concerned with the volume of fuel and volume of air than with their weight. It may not be necessary to enter on this matter at this stage, but the main principles will be briefly indicated. We will choose the combustion of carbon monoxide, because no question of condensation of the resulting product arises at about atmospheric temperature and pressure (as it does with the steam formed by burning hydrogen). The equation $2CO+O_2=2CO_2$, not only implies that $2(12+16)$ or 56 lb. of CO combine with 2×16 or 32 lb. of oxygen to form 2×44 or 88 lb. of CO_2. It also implies that 2 molecules of CO combine with 1 molecule of oxygen to form 2 molecules of CO_2. And here we use Avagadro's Law, arrived at by experiment, but also deducible from the mechanics of the ideal gas. This law may be stated thus; at the same temperature and pressure equal volumes of all gases contain equal numbers of molecules. Hence to provide the 1 molecule of oxygen for every 2 molecules of CO required by the foregoing equation, we require 1 cu. ft. of oxygen for every 2 cu. ft. of carbon monoxide (CO), both being at the same temperature and pressure. And the product from the total of 3 cu. ft. of gases before combination, will be only 2 cu. ft. of the product CO_2 when the temperature and pressure have reached their original values. If the reaction is in a closed vessel of 3 cu. ft. capacity the pressure would only be two-thirds of the original after the temperature had fallen to its original level. Then if the CO_2 be pressed into a space of 2 cu. ft. at constant temperature, the pressure (following Boyle's Law) would rise to just the initial pressure and the product would be 2 cu. ft. of CO_2 at the same temperature and pressure as the original gases.

Similarly for the burning of hydrogen, the equation

$$2H_2+O_2=2H_2O$$

implies that 2 cu. ft. of hydrogen combines with 1 cu ft. of oxygen to form 2 cu. ft. of water vapour at the original temperature and pressure, but unless that temperature be high enough, much of the water vapour (steam) may be condensed to liquid water.

Similarly for marsh gas CH_4,

$$CH_4+2O_2=CO_2+2H_2O$$

implies that 1 cu. ft. of CH_4 combines with 2 cu. ft. of oxygen to form 1 cu. ft. of CO_2 and 2 cu. ft. of steam.

For mixtures of gases given in percentages *by volume*, the volume of oxygen required is easily found by taking the appropriate fractions which the percentages represent of 1 cu. ft. of mixed gases. And since air contains about 21 per cent. of oxygen by volume, the amount of air to give 1 cu. ft. of oxygen is $\frac{100}{21}$ or 4·76 cu. ft., or the minimum volume of air required is 4·76 times the volume of oxygen required by the above equations.

Example 4. Find the minimum number of cubic feet of air necessary to burn 1 cu. ft. of gas containing by volume 40 per cent. of hydrogen, 50 per cent. of marsh gas and 10 per cent. of carbon monoxide.

0·4	cu. ft. of	H	requires	$\frac{1}{2}$ of 0·4	=0·2	cu. ft. of oxygen
0·5	,, ,,	CH_4	,,	$2\times0·5$	=1·0	,, ,, ,,
0·1	,, ,,	CO	,,	$\frac{1}{2}$ of 0·1	=0·05	,, ,, ,,

adding, 1·0 cu. ft. of mixture requires 1·25 cu. ft. of oxygen, and 1·25 cu. ft. of oxygen will require $\frac{100}{21}\times1·25=5·95$ cu. ft. of air.

82. Calorific Values of Fuels. If a fuel were made up of constituent parts in known proportions and if these constituents parts were in no way chemically combined, the calorific value, or heat given out by a pound of such fuel in combustion could be calculated from the calorific values of its constituents. But, in fact, the constituent elements in a solid or liquid fuel are often in some measure combined and the extent of combination may be unknown and thus there is an uncertainty about calculations of calorific value of fuels from the calorific values of their constituent chemical elements. But the calculation is best explained by an example.

Example. A coal contains 80 per cent. carbon and 5 per cent. of hydrogen, and the remaining 15 per cent. is incombustible (ash, etc.). Find the calorific value of the coal.

In Art. 80 the calorific value of carbon was given as about 14,540 B.Th.U. per lb., or 8080 lb.-C.H.U. per lb., and that of hydrogen as 60,930 B.Th.U. per lb. or 33,830 lb.-C.H.U. per lb. But as 1 lb. of hydrogen produces 9 lb. of steam, which carries about $9 \times 970 = 8730$ B.Th.U. of latent heat, it is usual to deduct this, leaving $60{,}930 - 8730 = 52{,}200$ B.Th.U. per lb. as the *lower* calorific value of hydrogen, while 60,930 B.Th.U. per lb. is called the *higher* calorific value. The corresponding values for the centigrade scale are for hydrogen, higher value 33,830 lb.-C.H.U. per lb., and lower value 29,000 lb.-C.H.U. per lb.

Adopting the lower calorific value for hydrogen, in the coal having 80 per cent. C and 5 per cent. H, per lb. burnt.

Heat given by 0·80 lb. C $= 14{,}540 \times 0{\cdot}8 = 11{,}632$ B.Th.U.
Heat given by 0·05 lb. H $= 52{,}200 \times 0{\cdot}05 = 2{,}610$ B.Th.U.

Total heat from 1 lb. of coal $= 14{,}242$ B.Th.U.

The outstanding doubt here is whether the hydrogen is free from prior combination with oxygen in the coal; part of the incombustible 15 per cent. may be oxygen, and in combination with the hydrogen. Since oxygen in water is only combined with one-eighth of its weight of hydrogen, sometimes in such calculations, the amount of hydrogen is reduced by one-eighth of the weight of oxygen present, if this is known. But this is a speculative refinement and calculations of this kind are at best an approximation.

Fuel Calorimetry. Instead of relying on the rather doubtful type of calculation exemplified above, and needing an analysis of the fuel, the calorific value of a fuel is determined by direct experiment in a fuel calorimeter. For a determination of the calorific value of a solid or liquid fuel, a weighed sample is burned in oxygen in a vessel submerged in a water, the water equivalent of the water and containers being known. From a measurement of the temperatures before and after burning, the heat given out by the known weight of fuel, and hence the calorific value of the fuel, can be calculated. Frequently a calorimeter of the " bomb " type is used. In this the weighed fuel is placed in a crucible which is enclosed in a strong gas-tight metal vessel or bomb into which compressed oxygen is forced before the charge is ignited by means of an electrical fuse wire. But fuel calorimeters take various forms and should be examined and their use demonstrated in a laboratory.

83. Steam Boilers. A boiler, with its fire-grate and flues, provides for the combustion of fuel and for the transfer of the heat liberated by combustion from the fuel to the water or steam in the boiler. Some of the heat will pass to the metal walls of the furnace by radiation, but some of it will be carried by the furnace gases to the metal heating surface of the boiler beyond the actual furnace chamber or fire box. In this way the furnace gases may be cooled to more nearly the temperature of the steam in the boiler, thus allowing the heat to be more completely used than if the gases passed directly from the furnace to the atmosphere without some circulation beyond the combustion chamber.

The strongest shape of metal envelope to withstand internal pressure would be a spherical shell, but this is not a developable surface. It has curvature in all directions and cannot be formed by bending a flat plate between cylindrical rollers. Consequently boilers generally consist of, or include, a cylindrical shell or drum. The curved surface of this has no tendency to bulge or buckle under the action of internal pressure and the circular ends, if made plane, and other flat surfaces of combustion chambers, exposed to pressure, which would be distorted if unsupported, are restrained by some form of stays. Boilers vary widely in form according to circumstances which go to determine the shape and proportions in different situations. They may be broadly divided into two classes (*a*) smoke tube or fire tube, and (*b*) water-tube boilers. In the former class, the furnace is generally inside the boiler within a metal firebox which is virtually an integral part of the boiler, and the furnaces gases pass through either large or small tubes inside the boiler, and in some cases along flues which are bounded by the external shell and the brickwork setting. The furnace of a water-tube boiler is external and the water circulates inside tubes which are exposed, some parts to the fire and others to hot gases which have left the furnace.

84. Water-Tube Boilers. The seamless steel tube or the cylinder of small diameter is a strong form to resist internal pressure, and water-tube boilers need employ as reservoirs of steam and water only cylindrical drums which are small relatively to the capacity of the boilers for raising steam, while the tubes being of small diameter can be made much thinner than a boiler shell. A diagrammatic representation of a water-tube boiler with straight tubes, a small Babcock land boiler, is shown in Fig. 73. The water tubes placed in inclined positions promote rapid circulation of the water, since hotter water is lighter than water which is less hot. Hotter water and steam will be flowing from the higher end of the tube into the horizontal cylindrical drum, and cooler water will be flowing down the steeply inclined tubes from the drum into the lower end of the main tubes. The amount of heat which can be passed through the metal of a boiler depends not only on the difference of temperature on the two sides of it, but also on the speed at which, on the wet side, heated water makes way for cooler water and, on the dry side, gases which have given up some of their heat make way for hotter gases. In other words, the performance of a boiler depends upon the rate of circulation of the water within it, and of the speed of the flue gases, and in this the water-tube boiler is outstanding. The demand for high-pressure steam and for the rapid production of steam have helped to make the water-tube type of boiler very widely used for marine service where lightness and small size in relation to steam raising capacity, and the possibility of erection in a confined space are also important. And the need for high-pressure steam on account of its greater economy in the conversion of heat into mechanical energy in engines or turbines, has also led to the use of water-tube boilers in large power plants.

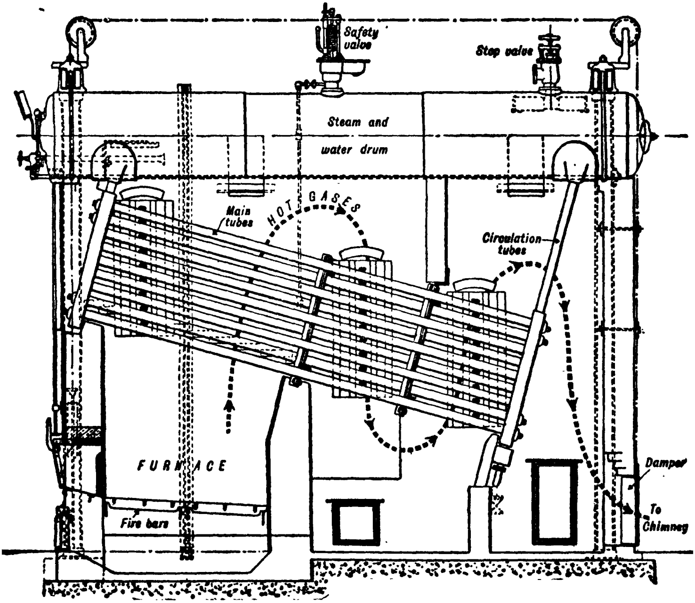


Fig. 73.—Small Babcock water-tube boiler.

The flow of the flue gases indicated by arrows in Fig. 73 is controlled or guided by the partitions. The flames and hottest gases impinge on the upper end of the sloping tube containing the hottest water, while the coolest water is exposed at the lower end of the tube, to the gases where they are coolest, just before they pass to the chimney flue. This type of water-tube boiler has straight and relatively large tubes convenient for cleaning and replacements. In some other types, smaller tubes, sometimes curved, and more than one steam drum are used. Generally water-tube boilers occupy a relatively small floor space and require considerable space vertically, conditions which are advantageous on some sites, but not in all situations, e.g. for steam locomotion purposes, where height is severely restricted.

85. Draught. The rate of movement of the air through a boiler fire and of the furnace gases over the heating surface of the boiler as they pass towards the chimney, affects the amount of heat which can be passed through the heating surface as already stated in the preceding article. But it also greatly affects the rate at which fuel can be burnt and thus the amount of heat available and hence, in another way, the steam-raising capacity of the boiler. It is common knowledge that blowing a fire makes it burn more vigorously ; the air in contact with the burning fuel, after giving up most of its oxygen, only impedes combustion by blocking the way to contact of fresh air with the fire, unless there is a draught to clear away the inert gases and bring into action fresh supplies of oxygen. Thus much more fuel can be burned and more steam can be raised by the use of a strong draught. " Natural " draught is due to the difference in weight of the hot and lighter gases in a chimney and the cooler and heavier air outside which produces a greater pressure under the fire bars of the furnace grate than there is in the chimney. The pressure difference increases with the height of the chimney and also with the temperature of the hot gases in the chimney. But it is wasteful to allow the flue gases to go up the chimney while still containing any heat which can be extracted from them. And a high chimney is very costly. Thus when a high evaporative performance is required, some form of "forced draught" produced by mechanical means, such as a fan or a blower, is employed. Moreover, experience shows that with a high draught it is possible to burn coal with less air per pound of fuel than with a smaller draught such as is produced by "natural" means. And thus with a smaller weight of furnace gases per pound of fuel carrying away heat to the chimney, less heat is lost and a larger proportion goes into the steam produced, provided that the heating surface is made adequate to reduce the flue gases sufficiently in temperature. With forced draught it is possible to burn poor grades of fuel which would not burn satisfactorily with natural draught.

In some cases, instead of forcing air through the fire, it is *drawn* through by means of a fan placed between the boiler and the chimney. The draught is then said to be *induced* instead of *forced*. A boiler may have both forced and induced draught.

Example. Find the heat lost in the flue gases (*a*) with natural draught, allowing 24 lb. of air per lb. of fuel, and (*b*) with forced draught, taking 18 lb. of air per lb. of fuel, if in each case the gases leave the flue at 590° F. and the outside air is at 60° F. Take the specific heat of the gases as 0·24.

(*a*) Total weight of gases per pound of fuel $= 24 + 1 = 25$ lb. Excess of temperature over atmospheric $= 590 - 60 = 530°$ F. Heat lost per pound of fuel $= 25 \times 0{\cdot}24 \times 530 = 3180$ B.Th.U.

(*b*) Total weight of gases per pound of fuel $= 18 + 1 = 19$ lb. Excess of temperature over atmospheric $= 590 - 60 = 530°$ F. Heat lost per pound of fuel $= 19 \times {\cdot}024 \times 530 = 2417$ B.Th.U.

Air Preheating and Economisers. The main loss of heat in connection with a boiler is that carried away to the chimney by the gases which go out so much hotter than the air (of which they mainly consist) comes into the furnace. Various means of reducing this loss are employed. The cold feed-water may be circulated in pipes surrounded by the waste gases. Such an apparatus is called an economiser. And the air before entering the furnace may also be warmed by the waste heat of the gases: this is called air preheating.

86. Smoke-Tube or Fire-Tube Boilers. There are very many types of smoke-tube boilers: some have one or two large tubes and others have a large number of smaller tubes giving a large heating surface but offering a considerable resistance to the passage of the hot gases through them. They are called multitubular boilers and are commonly used in association with some form of forced draught. We illustrate diagrammatically and briefly describe two multitubular boilers and two large-tube boilers. No attempt is made to show the detailed form, which is dealt with in books and classes on construction. The flat surfaces exposed to steam pressure are in all cases stayed by various appropriate means. In addition to these diagrams, which are simplified to facilitate explanation, the reader should see actual boilers and detailed drawings.

The Lancashire Boiler. This is illustrated diagrammatically in Fig. 74. It consists of a cylindrical shell with two large tubes in which are the two furnaces at the front end of the boiler. The brickwork setting not only supports the boiler but

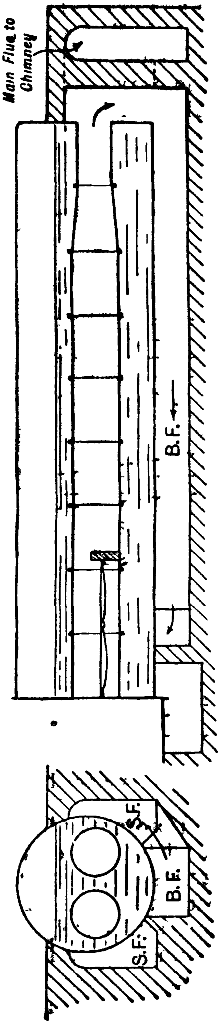


FIG. 74.—Lancashire boiler.

forms an essential part of the heating arrangement by forming flues. The hot gases from the fires pass over the furnace bridge and along the two tubes to the back end of the boiler, giving up a large proportion of their heat on the way. Here they pass into a common bottom flue BF and pass along under the boiler, heating the water through the external shell. Near the front of the boiler the gases divide into two streams and pass upwards into the two side flues SF. In these they traverse once more the length of the boiler and into a flue leading to the chimney. This flue has dampers, in the form of plates, to control the flow. These plates can be lifted or lowered in slots by means of chains or wire ropes, which can be pulled or released from the front of the boiler. Lowering the damper checks the flow and checks the rate of combustion of coal in the furnace.

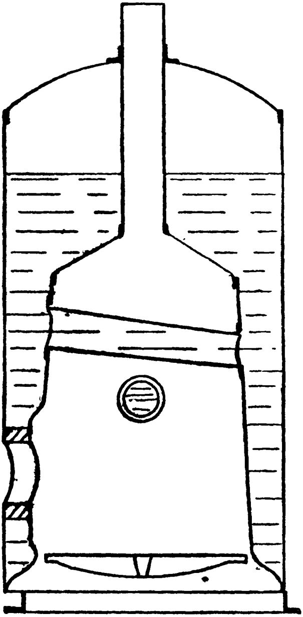


Fig. 75.—Vertical boiler.

The Lancashire boiler is about 30 ft. long and is not made of smaller diameter than about 8 ft. since it has two tubes each of which must be large enough to accommodate a furnace of workable size. If a smaller boiler of this general type is required it is made with a single tube and is then called a Cornish boiler.

Lancashire boilers for long held a predominant position for mills and large power installations on land and are still widely used and in many cases with natural or chimney draught only.

Vertical Boiler. Fig. 75 shows a vertical cylindrical boiler with one large central internal tube or firebox containing the furnace and merging into a smaller central tube through which the still hot gases pass to the chimney. This boiler is used when the demand for steam is small. It cannot be so efficient as other types but it occupies only a small floor space and requires no foundations. The firebox is pierced by two or more water tubes connecting the water spaces on opposite sides of the firebox. These provide additional heating surface in contact with the hottest furnace gases and promote circulation of the water.

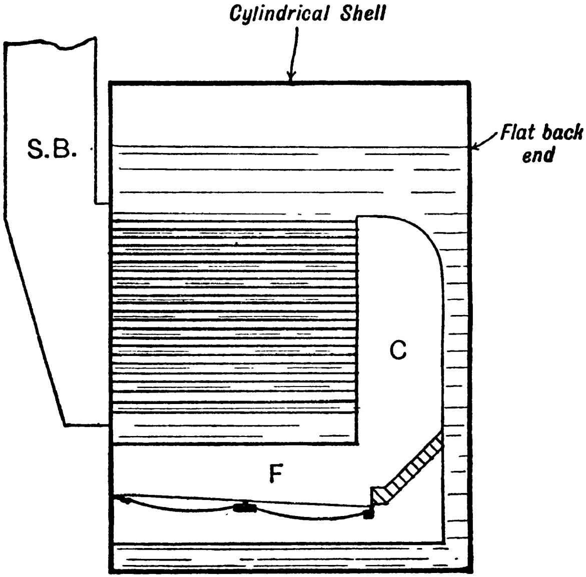


Fig. 76.—Scotch marine boiler.

The Scotch Marine Boiler. This cylindrical boiler, shown diagrammatically in Fig. 76, is relatively short but large in diameter, for it may be 15 ft. or more in diameter and 10 ft. or more in length. There are generally three furnaces F, and just beyond them the gases pass into a common combustion chamber C.

Between this and the back flat end of the boiler, the water circulates. The hot gases pass from C to the front of the boiler again by a large number of "smoke tubes," which make this a "multitubular" type of boiler. From the tubes the gases pass upwards through the smoke box SB to the chimney or funnel. These boilers were formerly in general use for marine work but have, to a considerable extent, been replaced by water-tube boilers. Where a number of Scotch marine type boilers are required, weight and cost are often reduced by making them "double ended." These double-ended boilers are about twice as long as single-ended boilers and have furnaces at each end leading into either a common combustion chamber at the middle of the boiler or into two independent ones separated by a water space. A land boiler somewhat resembling the Scotch marine boiler and called the "dryback" or "economic" boiler, instead of an internal metal combustion chamber has an external one built of brickwork or of metal plate lined with firebrick.

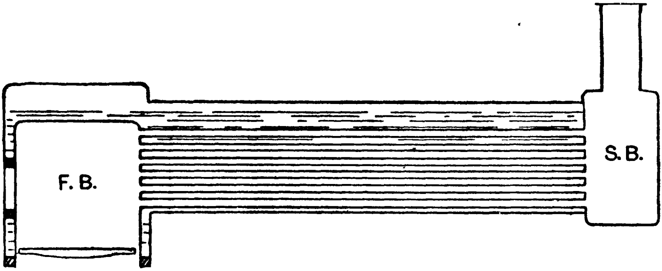


FIG. 77.—Locomotive type boiler.

Locomotive Boiler. This boiler, the form of which is so obviously determined by its special circumstances, is shown diagrammatically in Fig. 77. On a railway, the head room is very limited and consequently a cylindrical boiler of relatively small diameter and great length became inevitable as soon as locomotives of great power were built. The furnace is in an internal firebox and the hot gases pass through long tubes to the smoke box and up the chimney, which in modern locomotives has become shorter as boiler diameters have become larger. The use of such long small-diameter tubes is made possible by the strong upward draught induced by the jet of exhaust steam, liberated from the engine cylinders under a pressure considerably above that of the atmosphere. Under this draught the amount of fuel which can be burned is large in relation to the area of the grate and (assisted perhaps by vibration in freeing the smoke tubes from external bubbles) the amount of steam raised is large although the efficiency is not high.

87. Performance of Boilers. The function of a boiler being to transfer the energy in the coal to the steam, as heat energy, its performance may be judged by the amount of heat in the steam produced, in relation to the heat obtainable from the fuel burned in producing the steam. The input of energy, from the weight of fuel burned and its calorific value, and the output from the weight of steam formed and its heat per pound might be measured over an hour, a day, a week, or a year. But it is little satisfaction to have a boiler which works well for, say, 8 hours in a day if in the other 16 hours it uses much fuel for no useful purpose, that is, if there are large stand-by losses. The necessity of making and maintaining a boiler fire when not in active use and of skilled attention, is one of the disadvantages of a steam power plant for intermittent use. And it is difficult to measure fuel consumption over a short period such as an hour, because, while the fuel fed on the grate can easily be measured, in order to know the consumption, it would be necessary to know whether the fuel on the grate and partly burnt was the same in quantity and condition at the beginning and end of the period. These difficulties are generally reduced by making a test or trial of a boiler over a long period. Assuming the steam output, its pressure and dryness, and the fuel consumed to have been measured, the ratio of the total amount of heat added to the feed water, to the heat obtainable from the fuel consumed, is called the efficiency of the boiler or

Boiler efficiency

$$= \frac{\text{Heat given to steam produced}}{\text{Heat in fuel consumed}}$$

$$= \frac{(\text{pounds of steam produced}) \times (\text{increase of heat per lb.})}{(\text{pounds of fuel consumed}) \times (\text{calorific value per lb.})}$$

Equivalent Evaporation. Sometimes the performance of a boiler is expressed by giving the number of pounds it produces per hour. This is a measure of the steam-raising capacity without regard to whether it is wasteful or not. And again, sometimes the performance is stated in pounds of steam raised per pound of coal burned. But as the heat transferred to the steam varies with the temperature (or pressure) and the dryness of the steam and also according to the temperature of the feed water entering the boiler, it is common to state, not the actual number of pounds of steam raised but the equivalent number of pounds evaporated under some standard condition. And that standard is steam at atmospheric pressure from water at 212° F. which requires 971 B.Th.U. per lb. (the latent heat of steam at 212° F.).

Thus, if a boiler produced 6000 lb. of dry steam per hour at 100 lb. per sq. in. from feed water supplied at 102° F., the heat given to the steam per pound would be the total heat in the steam per pound minus the sensible heat in the feed water per pound. And from the steam tables

Heat per pound at 100 lb. per sq. in. = 1191 B.Th.U. per lb.
Sensible heat in feed water at 102° F. = 70 B.Th.U. „ „

Heat supplied per pound = 1121 B.Th.U.

B.Th.U. supplied per hour = 6000 × 1121 B.Th.U.

$$\text{Equivalent evaporation ``from and at'' } 212^\circ\text{ F.} = 6000 \times \frac{1121}{971}$$

$$= 6928 \text{ lb. per hour.}$$

Example. A boiler uses 18,000 lb. of feed water per hour, the steam being at 200 lb. per sq. in. absolute and 0·96 dryness fraction. If the feed water is at 200° F. and the fuel consumed is 2200 lb. per hour, of calorific value 14,500 B.Th.U. per lb., find the boiler efficiency and the equivalent evaporation from and at 212° F. per hour and per pound of fuel burned.

From the steam tables, at 200 lb. per sq. in pressure

Latent heat of dry steam per pound = 848 B.Th.U.
Latent heat of steam 0·96 dry per lb. = 0·96 × 850 = 814 B.Th.U.
Sensible heat in water per pound = 355·5 B.Th.U.

Total heat per pound of wet steam = 1169·5 B.Th.U.

Sensible heat per pound in feed water = 200 − 32 = 168

Heat added per pound evaporated = 1001·5 B.Th.U.
Heat transferred to steam per hour = 18,000 lb. × 1001·5 B.Th.U. per lb.
= 18,027,000 B.Th.U.

Heat in fuel used per hour = 2200 lb. × 14,500 B.Th.U./lb.
= 31,900,000 B.Th.U.

Boiler efficiency $= \frac{18{,}027{,}000}{31{,}900{,}000}$
= 0·565 or 56·5 per cent.

Equivalent evaporation per hour from and at 212° F. $= \frac{18{,}027{,}000 \text{ B.Th.U./hr.}}{971 \text{B.Th.U./lb.}}$
= 18,570 lb./hr.

Equivalent evaporation from and at 212° F. per pound of coal $= \frac{18{,}570 \text{ lb. steam/hr.}}{2200 \text{ lb. of coal/hr.}}$
= 8·44 lb. of steam/lb. of coal.

Summary of Chapter VIII

Combustion is the chemical combination of fuel with oxygen, with which the constituents of the fuel unite in definite proportions. Atmospheric nitrogen dilutes the resulting gases and keeps the temperature from rising to the extent it would do for combustion in oxygen only. Hydrogen, which forms water vapour, and carbon, which forms CO_2, and other constituents in burning give out definite amounts of heat according to the weight present, but the calorific value of any fuel is best determined by the direct method of burning a known weight of it in a fuel calorimeter.

The characteristic features of five different kinds of boilers in wide use are described ; each type has its own special field of application.

The efficiency of a boiler is represented by the ratio of the heat which it gives to feed water in converting it into steam, to that available in the fuel consumed. The performance is estimated in terms of the equivalent evaporation in pounds of steam from feed water at 212° F. to dry saturated steam at 212° F. per hour.

EXAMPLES VIII

1. A fuel contains 85 per cent. of carbon, 5 per cent. of hydrogen, and the remaining 10 per cent. is incombustible. How many pounds of air would be required for its combustion if an excess of 50 per cent. above the minimum to supply the necessary oxygen for combustion, is allowed ?

2. What would be the percentage by weight of carbon dioxide, nitrogen, and oxygen respectively in the gases leaving the furnace for the data in No. 1, assuming that combustion is complete and that air contains 23 per cent. by weight of oxygen ?

3. Find the minimum volume of air necessary to burn a cubic foot of gas consisting, by volume, of 60 per cent. of marsh gas (CH_4) and 40 per cent. of carbon monoxide (CO). Take air as containing 21 per cent., by volume, of oxygen.

4. Find the calorific value of the fuel in No. 1 if that of carbon is 14,450 B.Th.U. per lb. and the lower calorific value of hydrogen is 52,200 B.Th.U. per lb.

5. In a calorimeter test, 1·05 gm. of coal is burnt in a calorimeter containing 2,000 gm. of water and having a water equivalent of 680 gm. If the rise in temperature of the water and calorimeter is 5·49° F., find the calorific value of the coal in B.Th.U. per pound and in lb.-C.H.U. per pound.

6. A boiler generates 6,000 lb. of steam per hour at 180 lb. per sq. in. and 0·95 dry, from feed-water at 92° F. If it uses 740 lb. of coal per hour of calorific value 14,000 B.Th.U. per lb., find the boiler efficiency and the equivalent evaporation from and at 212° F. per lb. of coal.

7. What weight of coal will be required per hour to produce 20,000 lb. of dry saturated steam per hour at 180 lb. per sq. in. from feed-water at 100° F. if the calorific value of the coal is 14,000 B.Th.U. per lb. and boiler efficiency is taken as 70 per cent. ?

ANSWERS TO EXAMPLES

EXAMPLES I (p. 27)

1. 212·7 m.p.h.; 312 ft. per sec.; 16·9 sec.
2. 31·4 ft. per sec. **3.** 23·25 metres per sec.
4. 5·5 sec.; 121 ft. **5.** 1·5 ft. per sec. per sec.
6. 23·6 radians per sec. **7.** 32·6 radians per sec.
8. 0·67 m.p.h. per sec.; 0·978 ft. per sec. per sec.; 60 sec.
9. 9·09 sec.; 4·84 ft. per sec. per sec.
10. 8 r.p.m. per sec.; 0·838 radian per sec. per sec.
11. 43·6 sec.; 0·048 radian per sec. per sec.
12. 37·3 ft. per sec. per sec.; 30·4° N. of W.
13. 4·5 ft. per sec.; 14·6 ft. per sec.; 11·4 ft. per sec.
15. 880 ft.

EXAMPLES II (p. 65)

1. 204 lb. **2.** 91·1 tons.
3. 1·15 ft. per sec. per sec.; 517·5 ft.; 4·64 h.p.
4. 18,400 lb.; 1,963 h.p.
5. 27·4 h.p. **6.** 67·0 lb.
7. 1·202 ft. per sec. per sec.; 18·7 lb.
8. 36,000 lb.; 93·1 ft. per sec.
9. 133·3 revolutions. **10.** 133 seconds.
11. 262 lb.-ft.; 98,880 ft.-lb. **12.** 1,915 lb.
13. 732 lb.; 18·7 lb. **14.** 1,144 lb.; 44·8 lb.
15. 33·3 lb.; 21·8 lb.

EXAMPLES III (p. 93)

1. 89,800 ft.-lb.; 1,497 ft.; 706 ft.
2. 2,000 joules. **3.** 9,506 ft.-lb.
4. 45 ft.-lb.; 13·9 ft. per sec.; 1 ft.
5. 8·3 ft. per sec.; 11·35 ft. per sec.
6. 15 ft. per sec.
7. (*a*) 8 ft. per sec.; (*b*) 8·75 ft.-lb.; 5·31 ft. per sec.
(*c*) 11·25 ft.-lb.
8. 12·65 ft. per sec.; 10·34 lb.
9. 197,700 ft.-lb. **10.** 90·6 r.p.m.
11. 300 ft.-lb. **12.** 9,270 lb.
13. 31,930 ft.-lb.; 122,700 lb.
14. 497,500 ft.-lb. **15.** 0·476 ft.

EXAMPLES IV (p. 131)

1. 19·64 lb. per sq. in. 2. 1,520,000 lb.
3. 5,489 lb.
4. 10,000 lb.; 8,351 lb.; 5,567 lb.
5. 33,600 lb. 6. 153·2 gal.
7. 1,361 lb.; 1·94 h.p.; 20·7 gal.
8. 20·8 amp.; 4·16 pence.
9. 338 cu. ft. 10. 0·257 inch.
11. 71·8 ft. per sec.; 1,488 lb. per sq. ft.
12. 613 lb. per sq. ft.
13. 423 lb.; 868 lb.

EXAMPLES V (p. 151)

3. 60 lb. per sq. in.; 12,000 ft.-lb.
4. 363·5 lb. per sq. in. 5. 473·3° F.
6. 18·6 per cent. 7. 124 cu. ft.
8. 1·323 lb.; 17,420 ft.-lb.
9. 4,884 ft.-lb.; 21·76 B.Th.U.; 0·169.

EXAMPLES VI (p. 185)

1. 63,780 ft.-lb.; 810·2 B.Th.U.
2. 67·2 lb. per sq. in.
3. 30·0 h.p.
4. 118·6 lb. per sq. in.; 10 h.p.; 0·847 pints per h.p.-hour; 0·783 lb. per h.p.-hour; 16·7 per cent.
5. 111·6 lb. per sq. in.
6. 8 in.; 12 in.
7. 106,300 ft.-lb.

EXAMPLES VII (p. 222)

1. 36·1 lb. 2. 41,220 lb. per hr.
3. 7·43 b.h.p. 4. 17·15 per cent.
5. 45·9 lb. per sq. in.

EXAMPLES VIII (p. 246)

1. 17·4 lb.
2. 17·0, 73·2, 7·3 per cent.
3. 6·67 cu. ft.
4. 14,892 B.Th.U. per lb.
5. 14,020 B.Th.U. per lb.; 7,789 lb.-C.H.U. per lb.
6. 63·6 per cent.; 9·18 lb.
7. 2,314 lb. per hour.

PROPERTIES OF SATURATED STEAM

Compiled mainly from the " Revised Callendar Steam Tables " (1931) (B.E.A.M.A.) and supplemented for pressures below 1 lb. per sq. in. by values from the Abridged Callendar Steam Tables, published by Messrs. Edward Arnold & Co., by whose permission they are reproduced.

Pressure lb./in.^{2} abs. p	Temperature ° F. t	Sp. vol. ft.^{3}/lb. V	Heat in B.Th.U.			Heat in lb.-C.H.U.		Temperature ° C.
			Sensible h	Latent L	Total H	Sensible h	Total H	
0·2	53·0	1524·0	21·01	1058·65	1079·66	11·7	599·8	11·7
0·4	72·8	790·7	40·69	1048·26	1088·95	22·6	605·0	22·7
0·5	79·5	640·5	47·42	1044·70	1092·12	26·3	606·7	26·4
0·6	85·2	539·1	53·03	1041·71	1094·74	29·5	608·2	29·5
0·8	94·4	411·1	62·19	1036·80	1098·99	34·6	610·6	34·7
1·0	101·7	333·0	69·5	1032·9	1102·4	38·6	612·4	38·7
2·0	126·1	173·5	93·9	1019·7	1113·6	52·2	618·7	52·3
4·0	153·0	90·52	120·8	1004·9	1125·7	67·1	625·4	67·2
6·0	170·1	61·89	137·8	995·4	1133·2	76·6	629·6	76·7
8·0	182·9	47·26	150·7	987·9	1138·6	83·7	632·6	83·8
10·0	193·2	38·37	161·1	981·9	1143·0	89·5	635·0	89·6
14·7	212·0	26·77	180·0	970·7	1150·7	100·0	639·3	100·0
15	213·0	26·25	181·0	970·2	1151·2	100·6	639·6	100·6
18	222·4	22·14	190·5	964·4	1154·9	105·8	641·6	105·8
20	228·0	20·06	196·1	961·0	1157·1	108·9	642·8	108·9
30	250·3	13·72	218·8	946·7	1165·5	121·6	647·5	121·3
40	267·2	10·48	236·0	935·6	1171·6	131·1	650·9	130·7
50	280·9	8·50	250·0	926·3	1176·3	138·9	653·6	138·3
60	292·6	7·162	262·0	918·1	1180·1	145·6	655·6	144·8
70	302·8	6·196	272·4	910·9	1183·3	151·3	657·4	150·4
80	311·9	5·466	281·9	904·2	1186·1	156·6	658·9	155·5
90	320·2	4·891	290·5	898·0	1188·5	161·4	660·3	160·1
100	327·9	4·429	298·5	892·2	1190·7	165·8	661·5	164·4
110	334·8	4·043	305·7	886·9	1192·6	169·8	662·6	168·2
120	341·3	3·727	312·5	881·8	1194·3	173·6	663·5	171·8
130	347·3	3·455	318·8	877·0	1195·8	177·1	664·3	175·2
140	353·0	3·221	324·9	872·3	1197·2	180·5	665·1	178·8
150	358·4	3·016	330·6	867·9	1198·5	183·7	665·8	181·3
160	363·6	2·836	336·0	863·7	1199·7	186·7	666·5	184·2
170	368·4	2·678	341·2	859·5	1200·7	189·6	667·1	186·9
180	373·1	2·536	346·1	855·6	1201·7	192·3	667·6	189·5
190	377·5	2·409	350·9	851·8	1202·7	194·9	668·2	191·9
200	381·8	2·293	355·5	848·0	1203·5	197·5	668·6	194·3
220	389·9	2·093	364·2	840·9	1205·1	202·3	669·5	198·8
240	397·4	1·926	372·2	834·2	1206·4	206·8	670·2	203·0
260	404·5	1·783	380·1	827·4	1207·5	211·2	670·8	206·9
280	411·1	1·660	387·3	821·2	1208·5	215·2	671·4	210·6
300	417·4	1·552	394·2	815·2	1209·4	219·0	671·9	214·1
350	431·8	1·336	410·1	801·0	1211·1	227·8	672·8	222·1
400	444·7	1·172	424·6	787·5	1212·1	235·9	673·4	229·8
500	467·1	0·940	450·1	763·1	1213·2	250·1	674·0	241·7
1000	544·7	0·458	544·1	660·2	1204·3	302·3	669·1	284·8
1200	567·4	0·373	574·0	622·8	1196·8	318·9	664·9	297·4

MATHEMATICAL CONSTANTS

Constant	Number	Log	Reciprocal	Log reciprocal
π	3·1416	0·4972	0·3183	$\bar{1}$·5029
$\pi/4$	0·7854	$\bar{1}$·8951	1·2732	0·1049

PHYSICAL CONSTANTS

Constant	No.	Log.
Acceleration due to gravity (g) ft. per sec. per sec.	32·2	1·5079
Acceleration due to gravity (g) cm. per sec. per sec.	981	2·9917
Density of air at N.T.P. in lb. per cu. ft.	0·0807	$\bar{2}$·9069
Volume of air at N.T.P. in cu. ft. per lb.	12·39	1·0931
Density of water at 60° F. in lb. per cu. ft.	62·4	1·7952
Density of water at 60° F. in lb. per gallon	10·0	1·0000
Specific heat of air at constant volume	0·1691	$\bar{1}$·2283

CONVERSION FACTORS

To convert	Multiply by	Log	Reciprocal	Log reciprocal
Radians to degrees	57·2978	1·7581	0·0175	$\bar{2}$·2418
Metres to inches	39·3704	1·5952	0·0254	$\bar{2}$·4048
Miles per hour to ft. per sec.	1·4667	0·1663	0·6819	$\bar{1}$·8337
Pounds to kilograms	0·4536	$\bar{1}$·6567	2·2046	0·3433
Pounds (force) to dynes	445,000	5·6484	$2{\cdot}247 \times 10^{-6}$	$\bar{6}$·3516
Horse-power to ft.-lb. per min.	33,000	4·5185	$3{\cdot}031 \times 10^{-5}$	$\bar{5}$·4815
Horse-power to ft.-lb. per sec.	550	2·7404	0·001818	$\bar{3}$·2596
Horse-power to kilowatts	0·746	$\bar{1}$·8727	1·341	0·1273
B.Th.U. to lb. C.H.U.	0·5556	$\bar{1}$·7448	1·800	0·2553
B.Th.U. to gram-calories	252·0	2·4014	0·003968	$\bar{3}$·5986
Therms to B.Th.U.	100,000	5·00	0·00001	$\bar{5}$·00
Foot-pounds to B.Th.U.	0·001285	$\bar{3}$·1090	778·0	2·8910
H.p.-hours to B.Th.U.	2545	3·4057	0·0003929	$\bar{4}$·5943
kWh. to B.Th.U.	3,409	3·5326	0·0002934	$\bar{4}$·4675
kWh. to therms	0·03409	$\bar{2}$·5326	29·34	1·4675
Foot-pounds to joules	1·3562	0·1323	0·7374	$\bar{1}$·8677
Joules to ergs	10^7	7·00	10^{-7}	$\bar{7}$·00
Joules to gram-calories	0·2392	$\bar{1}$·3788	4·18	0·6212

TRIGONOMETRICAL FUNCTIONS

Angle.		Chord	Sine	Tangent	Co-tangent	Cosine			
De-grees.	Radians								
0°	0	0	0	0	∞	1	1·414	1·5708	90°
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	89
2	·0349	·035	·0348	·0349	28·6363	·9994	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3007	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	85
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	83
8	·1396	·140	·1392	·1405	7·1154	·9903	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1763	5·6713	·9848	1·286	1·3963	80
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1·3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·218	1·3090	75
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	70
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	67
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	65
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	62
29	·5061	·501	·4848	·5543	1·8040	·8746	1·015	1·0647	61
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	60
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	59
32	·5585	·551	·5299	·6249	1·6003	·8480	·970	1·0123	58
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	55
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6458	·625	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	50
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	47
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	46
45°	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	45°
			Cosine	Co-tangent	Tangent	Sine	Chord	Radians	De-grees
								Angle.	

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	8	8	12	16	20	24	28	32	37
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934	0969					3	7	11	14	18	21	25	28	32
							1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	12	16	19	22	25	29
14	1461	1492	1523	1553							3	6	9	12	15	18	21	24	28
					1584	1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875	1903					3	6	9	11	14	17	20	23	26
							1931	1959	1987	2014	3	5	8	11	14	16	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	15	18	21	23
17	2304	2330	2355	2380	2405	2430					3	5	8	10	13	15	18	20	23
							2455	2480	2504	2529	2	5	7	10	12	15	17	19	22
18	2553	2577	2601	2525	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	6	5
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	3	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

ANTILOGS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTILOGS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	9
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

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